

California High School Exit Examination

Mathematics Teacher Guide 2002

Standards and Assessment Division
California Department of Education



California High School Exit Examination

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Introduction

The California High School Exit Examination (CAHSEE) Teacher Guides for English Language Arts (ELA) and Mathematics are designed to provide comprehensive and accessible information to assist teachers in preparing their students for the CAHSEE. This guide contains four sections, and teachers are encouraged to reproduce individual sections or all of the guide for classroom use. Districts and school personnel also are encouraged to use this material in their staff development activities.

- **Section 1**, “About the CAHSEE,” provides an overview of the purpose and content of the CAHSEE. This section also includes a set of checklists teachers may use when writing test questions for classroom use, information about security for the CAHSEE, and a glossary of terms used in this guide.
- **Section 2**, “CAHSEE Questions and Answers for Teachers,” provides answers to frequently asked questions about the administration and scoring of the CAHSEE.
- **Section 3**, “Assessing the CAHSEE Mathematics Standards,” is designed to give detailed information about how the California content standards for mathematics are tested on the CAHSEE. The mathematics standards are divided into six strands:

- Number Sense
- Statistics, Data Analysis, and Probability
- Algebra and Functions
- Measurement and Geometry
- Algebra 1
- Mathematical Reasoning

For each of these strands, section 3 provides a summary of the essential knowledge and skills covered in the strand, followed by a detailed description of each content standard within the strand. This discussion includes information about how the standard may be tested. Several CAHSEE released test questions are also discussed.

- **Section 4**, “Suggested Process for Using the CAHSEE to Increase Student Achievement,” details suggested processes for using the California content standards and the released test questions to help implement standards-based instruction in the classroom.

The **appendixes** to the Teacher Guide provide the following materials for easy reference:

- Appendix A: The CAHSEE mathematics blueprint (the number of test questions for each content standard)
- Appendix B: A worksheet and planning tool for using the CAHSEE to increase student achievement

Section 1

About the CAHSEE

Background

Education Code section 60850(a) required the State Superintendent of Public Instruction to develop a high school exit examination in language arts and mathematics in accordance with the statewide academically rigorous content standards adopted by the State Board of Education. Education Code section 60851 provides statutory authority to administer a high school exit examination and requires passage for obtaining a diploma.

Commencing with the 2003-04 school year, every graduating senior must pass the high school exit examination as a condition of receiving a high school diploma from a California public school. During the 2002-03 school year, students in grade 10 and those students in grade 11 who have not yet passed one or both parts of CAHSEE must take the test. In spring 2003, all students in grade 10 will be required to take the CAHSEE for the first time. For all students who do not pass, there will be multiple opportunities to retake one or both parts of the exam as necessary.

The State Superintendent of Public Instruction designates six testing dates a year for the CAHSEE. Testing dates are in March, May, July, September, November, and January. The specific dates are noted on the CDE website at <http://www.cde.ca.gov/statetests/cahsee>. Districts select from these dates and offer the CAHSEE at least three times a year (e.g., July, November, and March).

Purpose and Content of the CAHSEE

The purpose of the CAHSEE is to ensure that students who graduate from high school can demonstrate grade-level competency in the state content standards for reading, writing, and mathematics.

The State Board of Education has approved both grade level and specific content to be assessed on the CAHSEE. A High School Exit Examination Standards Panel, appointed by the State Superintendent of Public Instruction, reviewed the state content standards in mathematics and English language arts and recommended to the State Board which content standards students should master to receive a high school diploma. In mathematics, standards from grades 6, 7, and 8 were selected. In English language arts, the panel selected standards from grades 9 and 10 and two standards from grade 8.

The test questions on the CAHSEE assess the approved standards and cover a range of difficulty levels, consistent with good testing practices. Because the primary purpose of the examination is to determine whether or not students can demonstrate grade-level competency in reading, writing, and mathematics, many of the test questions are clustered around the difficulty level represented by the passing score. These questions assess full mastery of the designated California content standards as well as foundational knowledge and skills underlying these standards, as recommended by the High School Exit Examination Standards Panel.

For example, the ELA part of the CAHSEE includes test questions that require students to determine the meaning of words in context. This vocabulary strategy is not specifically named in the grade 9/10 content standards, but it is included on the exam because it is a foundational, underlying skill required for achievement of the grade 9/10 vocabulary standards. Similarly, most CAHSEE ELA test forms contain at least one poem. Although poetry is not specifically named in the grade 9/10 literary analysis standards, analysis of poetry is a focus of standards in the earlier grades. In mathematics, standards from grades 6, 7, and 8 are included on the CAHSEE because these grade level standards represent both foundational and competency standards that students should meet in order to graduate from high school.

All questions on the examination have been evaluated for their appropriateness for measuring the designated ELA and mathematics content standards. They have been reviewed and approved by committees of California educators, including teachers, administrators, and academicians. In addition to content, all items have been reviewed and approved by California educators for their adherence to the principles of fairness and have been evaluated to determine if bias exists with respect to characteristics such as gender, ethnicity, and language.

Both parts of the CAHSEE (ELA and Mathematics) have multiple-choice questions, which consist of a question or statement followed by a set of four possible answer choices. Only one answer choice is correct, and there is no scoring penalty for guessing an answer choice incorrectly. Mathematics items may include pictorial material, such as drawings, tables, diagrams, or graphs. There are two writing tasks on the ELA part of the CAHSEE. One task requires a written response to literary or informational text, and the other requires a written response to a stand-alone writing prompt.

Each operational form of the ELA and mathematics portions of the CAHSEE also includes 12 field-test questions consisting of newly developed items for which the collection of statistical data is necessary. The field-test questions are not identified in the test booklets. These questions are selected for try-out purposes only and are not included in any individual or group student score reports.

The test blueprints for the CAHSEE indicate which content standards are to be tested and the number of items per standard. More detailed information is available on the CAHSEE website (noted on the previous page in the Background section).

The CAHSEE is not a timed test, which means it has no fixed time limit in which students must complete the examination. However, students are expected to complete their work during the regular school day unless their Individual Educational Program (IEP) or 504 Plan specifies the need for extra time beyond the school day.

English Language Arts Content Standards

The standards for the English language arts part of the CAHSEE are taken from the California Content Standards for grades 9/10. Standards from the following strands are included: Word Analysis, Reading Comprehension, Literary Response and Analysis, Writing Strategies, Writing Conventions, and Writing Applications.

The English language arts part of the CAHSEE contains 82 multiple-choice test questions and 2 writing tasks distributed across the six strands, as shown in Table 1.

Table 1
Distribution of CAHSEE ELA Questions by Strand

Strand	Number of Multiple-Choice Items	Number of Writing Tasks
Word Analysis	10	-
Reading Comprehension	24	-
Literary Response and Analysis	24	-
Writing Strategies	11	-
Writing Conventions	13	-
Writing Applications	-	2
Total	82	2

Mathematics Content Standards

As mentioned above, the standards for the mathematics part of the CAHSEE are taken from the California Content Standards in grades 6, 7, and 8. They include standards from the following mathematical strands: Number Sense; Algebra and Functions; Measurement and Geometry; Statistics, Data Analysis, and Probability; Mathematical Reasoning; and Algebra 1.

The mathematics part of the CAHSEE contains 80 multiple-choice test questions distributed across the six strands as shown in Table 2.

Table 2
Distribution of CAHSEE Mathematics Questions by Strand

Strand	Number of Multiple-Choice Items
Statistics, Data Analysis, and Probability	12
Number Sense	14
Measurement and Geometry	17
Algebra and Functions	17
Mathematical Reasoning	8
Algebra 1	12
Total	80

Development of Test Questions for the CAHSEE

The test questions that appear on the CAHSEE have been through an extensive development process to ensure that they are valid and fair measures of what students know and are able to do.

Content Validity

To ensure that the CAHSEE is a valid measure of the specified content standards, the test questions are carefully designed to assess the content indicated in the test blueprints. Insofar as possible, each question requires students to demonstrate knowledge and/or skills in only one standard. Because many content standards cover a wide range of knowledge and skills, individual test questions may assess one component of the standard. Other questions may address underlying, foundational knowledge or skills that are required for higher achievement in the standard.

Technical Quality

Well-written test questions give students an opportunity to demonstrate what they know and are able to do; students do not have to “outthink” or “outguess” what the question is asking. When questions are clearly written and easily understood, students are able to provide evidence of their learning. Test questions have only one clearly correct answer. The language is simple, direct, and free of ambiguity. Questions should not be a test of reading ability or vocabulary if that is not their purpose. CAHSEE test questions are reviewed for content validity and technical quality by committees of California educators.

Fairness

Bias in testing can take several forms, including the use of unfamiliar or insensitive language and terms, the presentation of stereotypes, and the inclusion of concepts that are offensive or negative toward any group. During the development process, CAHSEE test questions are continually reviewed for potential bias to ensure that the CAHSEE meets the highest standards of fairness in testing.

The following checklist is used by CAHSEE item writers and review committees as a basis for evaluating the content validity, technical quality, and fairness of test questions. Teachers may also use this checklist to improve their own classroom assessments. For multiple-choice questions for either English language arts or mathematics, teachers may wish to write standards-based test questions to help students prepare for the CAHSEE. The checklist in Table 3 is provided so that teachers may evaluate their own questions against the general requirements for CAHSEE multiple-choice questions.

Table 3
Development Checklist for Multiple-Choice Questions

✓	Requirements
The test question as a whole—	
	Has one and only one clearly correct answer
	Clearly presents one central idea
	Measures the intended objective
	Has a clear purpose
	Is within the appropriate range of difficulty
	Contains simple, direct, and unambiguous language
	Uses age-appropriate vocabulary and sentence structure
	Does <u>not</u> use vocabulary and idiomatic phrases that could be unfamiliar
	Does <u>not</u> rely on students' possessing outside knowledge
	Tests worthwhile (not trivial or obscure) concepts or information
	Reflects current teaching practices
	Is <u>not</u> tricky or cute
	Does <u>not</u> appear to ask for the student's opinion
	Is grammatically correct
	Uses the active voice and avoids informal diction and usage
	Follows the appropriate style guidelines
	Is free of bias, sensitive language or topics, and stereotypes

Development Checklist for Multiple-Choice Questions, continued

✓	Requirements
The stem of the test question—	
	Gives the test taker a full sense of what the item is asking
	Is either a question or an incomplete statement
	Is both clear and concise
	If negative, contains no negatives in the distracters
The stimulus or passage for the test question(s)—	
	Is likely to be interesting to students
	Is correctly and clearly labeled
	Provides all the information needed to answer the questions
	Can be reproduced clearly in a test book
The response options—	
	Are written so that no one option is significantly different from the others in length, specificity, or complexity
	Relate to the stem in the same way
	Do <u>not</u> include an option that denies the truth of any other option
	Do <u>not</u> deny the truth of the stem
	Do <u>not</u> give clues to students, such as the use of absolutes like <i>always</i> and <i>never</i>
	Do <u>not</u> repeat words that could be placed in the stem
	Include plausible and reasonable misconceptions and errors
	Do <u>not</u> include distractors that are phrased differently but have the same meaning as other distractors

Test Security

One of the most significant guarantors of fairness to all students who take the CAHSEE is that passages, writing prompts, graphical materials, and test questions remain secure at all times. Individuals who circumvent or attempt to circumvent procedures to maintain test security diminish the legitimate and honest efforts of all other students and teachers to participate in the state's assessment system. The California Department of Education has the authority, according to the Education Code section 60851 (b) and (c) and the copyright statutes of the United States, to act against any individual or group of individuals who knowingly attempts to copy, duplicate, or transmit in any way, the contents of secure material from test booklets, answer documents, in whole or in part, to any other individual or group of individuals. The California Department of Education may employ procedures to maintain the test security of the CAHSEE, including but not limited to monitoring of test administration, document handling, and post-test analytic techniques such as mark discrimination analysis.

Resource Documents

The information in this Teacher Guide is based on the California content standards and the California frameworks in English language arts and mathematics. These documents may be ordered from the California Department of Education, or they may be downloaded from the CDE web site, as shown below:

The English Language Arts Content Standards for California Public Schools, Kindergarten Through Grade Twelve is available from the Publications Division, Sales Office, California Department of Education, P.O. Box 271, Sacramento, CA 95812-0271; 1-800-995-4099, ext. 6. It is also available at <http://www.cde.ca.gov/standards> on the Internet.

The Mathematics Content Standards for California Public Schools, Kindergarten Through Grade Twelve (1997) is available from the Publications Division, Sales Office, California Department of Education, P.O. Box 271, Sacramento, CA 95812-0271; 1-800-995-4099, ext. 6. It is also available at <http://www.cde.ca.gov/standards> on the Internet.

The Mathematics Framework for California Public Schools, Kindergarten Through Grade Twelve (1999), is available from the Publications Division, Sales Office, California Department of Education, P.O. Box 271, Sacramento, CA 95812-0271; 1-800-995-4099, ext. 6. It is also available at <http://www.cde.ca.gov/cdepress/math.pdf> on the Internet.

The Reading/Language Arts Framework for California Public Schools, Kindergarten Through Grade Twelve is available from the Publications Division, Sales Office, California Department of Education, P.O. Box 271, Sacramento, CA 95812-0271; 1-800-995-4099, ext. 6. It is also available at http://www.cde.ca.gov/cdepress/lang_arts.pdf on the Internet.

Glossary of Terms Used in this Guide

Answer Choices — The correct answer and the distracters in a multiple-choice test question.

Blueprint — The plan for assessment that specifies the number of questions on each test form according to strand and content standard.

Clueing — An instance in which one test question provides information that could be used to select the correct answer to another question, or an instance in which the stem in a multiple-choice question clues the correct answer.

Constructs — The underlying cognitive domains for each strand in the California content standards (e.g., conceptual understanding or problem-solving in mathematics; reading, understanding, and analyzing grade-level texts in English language arts).

Distractors — Incorrect answers to a multiple-choice stem.

Field Test Questions — Test questions that are administered to students to gain information about the quality of the question. Student performance on these questions does not affect student scores.

Foundational Knowledge/Foundational Skill — For the CAHSEE, knowledge or skill that a student would be taught and be expected to know prior to taking courses in mathematics and English language arts at grades 9 and 10.

Item — A test question written in one of several possible item formats.

Item Format — The basic design of a test question (e.g., multiple-choice, constructed-response).

Key — The correct answer to a multiple-choice question.

Multiple-Choice Question — A stem plus a number of response options or answer choices (four for CAHSEE).

Response Options — The choices in a multiple-choice question, consisting of one key (correct answer) and a number of distractors (three for CAHSEE).

Scoring Guide — The rubric or protocol to follow when assigning a point value to responses to a writing task.

Specifications — The document that includes a description of how each standard is assessed on the CAHSEE.

Standard — Statement of what students should know and be able to do.

Stem — The initial part of a multiple-choice test question in which the task or premise is given. The stem may be a question, an incomplete statement, or a set of directions.

Stimulus — A picture, graph, map, chart, quotation, or other text that students are asked to interpret when answering a test item.

Strand — A category of standards that relate to each other for purposes of reporting performance on the CAHSEE.

Section 2

CAHSEE Questions and Answers for Teachers

1. Who is eligible to take the California High School Exit Examination (CAHSEE)?

During the 2002-2003 school year, students in grade 11 (class of 2004) who have not yet passed one or both portions of the CAHSEE may continue to test on designated test dates selected by the school district. In spring 2003, all students in grade 10 (class of 2005) must take the CAHSEE for the first time.

2. How can teachers best prepare students for the CAHSEE?

It is important that teachers provide instruction in state content standards for English language arts and mathematics. Instructional quality and consistency will have a significant impact on the success of students who take the CAHSEE. In addition to providing instruction that meaningfully corresponds to the state content standards, teachers' dedication to helping students learn to the full extent of their abilities will ensure that students are successful on this assessment.

3. What happens if a student does not pass the CAHSEE?

Students who do not pass the CAHSEE in the spring of their 10th grade year may retake the exam up to seven times. Students retake only the part(s) of the exam not previously passed. The school district must provide additional instruction to assist students who do not pass the exam [Education Code section 60851(e)].

4. Are students with special needs required to take the CAHSEE?

Students with special needs must pass the CAHSEE to receive a high school diploma and must be allowed to take the CAHSEE with accommodations or modifications as specified in their Individualized Education Program (IEP) or Section 504 Plan for classroom or statewide testing. Students who take the test using a modification will receive an invalidated test score. However, if the invalidated test score is the equivalent to a passing score (i.e., 350 or higher), the district may submit a waiver request on behalf of the student to waive the requirement of successful passage of one or both parts of the CAHSEE to receive a diploma.

5. What if parents do not want their children to take the CAHSEE?

Commencing with the class of 2004, all students are required to pass the CAHSEE to receive a high school diploma from a California public school (Education Code Section 60851). Parents cannot exempt their children from this requirement if they want their children to obtain a high school diploma from a public school.

6. How much time will students have to work on the CAHSEE?

The CAHSEE is an untimed test, so students may continue working to a reasonable limit. While the examination is not timed, the administration of the ELA part of the exam is divided into two sessions, each approximately two hours long. The mathematics part of the exam is divided into two sessions, each approximately an hour and a half long.

It is important to note that untimed does not mean unlimited time. Reasonable limits on student testing time may be necessary to accommodate issues of schedules and availability of rooms. Every effort should be made to provide for students who are unable to finish within the suggested working times. If necessary, these students may require relocation to another room to continue their work.

Although all students may have extra time to complete the CAHSEE, students may have time beyond the regular school day to complete the exam only if their Individual Educational Program or Section 504 Plan specifies the need for such extra time.

7. What if a student is absent on testing days?

Districts must ensure that all 10th graders absent during the initial administration take the CAHSEE either on the next test date designated by the Superintendent of Public Instruction or on the next designated test date selected by the district. Students missing a testing date must wait until the next testing date to take the portion of the test affected by the absence.

8. What does the English language arts part of the CAHSEE cover?

The English language arts part of the CAHSEE addresses state content standards through grade 10. It consists of multiple-choice questions and two writing tasks. The reading part of the exam covers decoding, vocabulary, informational reading, and literary reading. This part includes 50 percent literary texts and 50 percent informational texts. The texts are grade appropriate and accessible to students in grade 10 who perform at a basic level. The writing part of the exam covers writing strategies, writing conventions, and writing applications.

9. What does the mathematics part of the CAHSEE cover?

The mathematics part of the CAHSEE addresses the state content standards through algebra 1 and includes number sense; algebra and functions; measurement and geometry; statistics, data analysis, and probability; algebra 1; and mathematical reasoning. Students must also demonstrate a strong foundation in computational skills and arithmetic, including the ability to work with decimals, fractions, and percents. The mathematics part of the exam consists of multiple-choice questions.

10. Will students be allowed to use calculators for the mathematics part of the CASHEE?

A calculator may be used only by those special needs students with an IEP or Section 504 Plan that specifies the use of a calculator.

11. Will formulas and conversion factors be provided on the mathematics part of the CAHSEE?

Formulas or conversions required to complete a test question will be provided. However, there will be no formulas provided for the perimeter of a polygon, the circumference of a circle, the area of a triangle or parallelogram (including a rectangle), or the volume of a rectangular solid. The formula for finding the area of a nontraditional figure like a rhombus will be provided.

12. Who determined the content of the CAHSEE?

A High School Exit Examination Standards Panel, appointed by the State Superintendent of Public Instruction, reviewed state content standards in English language arts and mathematics and identified the content standards students should master to receive a high school diploma. The identified content standards went through public review, and the State Board of Education adopted the CAHSEE test blueprints in December 2000.

13. How can teachers be involved in the development and implementation of the CAHSEE?

Teacher involvement is vital to the CAHSEE in many ways. Content review groups meet once or twice each year, usually for three days, to review test questions prior to field testing. Members of this committee must be approved by the California Department of Education. Teachers can apply for the content review committee by contacting CAHSEE support at 800-241-5687 or at cahsee-support@ets.org. Teachers are also needed as scorers of the constructed-response questions on the English language part of the CAHSEE. High school English teachers are especially encouraged to apply. Information and an application may be found at <http://www.ets.org/reader/own/cahseeown.html>.

14. Are CAHSEE test questions released to the public?

Appropriate test security policies preclude the disclosure of *any* test item from test forms currently in operational or field-test use. Actual test items will be seen only by students at the time of examination. However, approximately 120 test questions are released each year from previous test forms. There are 60 mathematics multiple-choice questions, 60 ELA multiple-choice questions, and 2 ELA writing tasks released annually. The released questions may be found on the CAHSEE website, <http://www.cde.ca.gov/statetests/cahsee>.

15. How are the individual student results reported?

The *Student and Parent Score Report* provides two types of information about a student's performance on both parts of the exam. On the front of the report, the left-hand side shows the student's scale score, the scale score required to pass, and the fact that the student passed or did not pass, for both the English language arts and the mathematics parts of the exam. The right-hand side of the front of the report shows the number of questions answered correctly for each major strand of the content standards in English language arts and mathematics tested with multiple-choice questions. Each written essay is reported with a score of 1 to 4, with 4 being the highest score students can earn. The back of the report provides interpretive information for students and their parents/guardians.

16. How should the individual student results be used?

It is important to recognize that a student's individual score on the CAHSEE fulfills the purpose for which the test was designed, which is to determine if a student has demonstrated grade-level competency in the California content standards to be eligible to receive a high school diploma. The CAHSEE is not designed as a diagnostic test that would indicate an individual student's specific strengths and weaknesses. The student's scores on each strand of the CAHSEE may, however, provide an indication of areas in which additional diagnostic information should be obtained and where remedial or supplemental instruction should be focused. Teachers should examine test results for their school and district to help them determine which strands and standards need additional emphasis in the classroom.

17. What is a scale score?

A scale score for the CAHSEE ranges from approximately 250 to 450. This type of score is used for reporting the CAHSEE results to provide a more precise measurement of a student's achievement and to assure that test forms given at different times are comparable in difficulty. The CAHSEE provides scale scores for individual students and a mean or average scale score for groups of students. An individual scale score of 350 is necessary to pass each part of the CAHSEE.

18. Who sees the CAHSEE results for individual students?

Only authorized school personnel, students, and parents/guardians see individual results. The school district must record on each student's permanent record whether or not the student passed each part of the exam.

19. What information is provided on the CAHSEE aggregate reports that are distributed to districts and also reported on the California Department of Education website?

Aggregate reports include CAHSEE results for schools, districts, counties, and the state and are posted annually in September. Results are reported for the following categories:

- all students tested
- gender
- ethnicity
- language fluency
- economic status
- special education program participation

Scores for English language arts and mathematics show:

- number of students tested
- number and percent of students who passed
- number and percent of students who did not pass
- mean (average) scale score

Additional scores for mathematics include the average percent correct for the following strands:

- number sense
- algebra and functions
- measurement and geometry
- statistics, data analysis, and probability
- algebra 1

Additional scores for English language arts include the average percent correct for:

Reading

- word analysis
- reading comprehension
- literary response and analysis

Writing

- writing strategies
- writing conventions

Writing Tasks

Aggregate reports provide an average score for each essay. The writing applications score counts for 30 percent of the English language arts score.

Section 3

Assessing the CAHSEE Mathematics Standards

The Mathematics part of the California High School Exit Examination (CAHSEE) assesses designated California content standards from grades 6 and 7 and Algebra 1. A multiple-choice format is used to assess six strands: Number Sense; Statistics, Data Analysis, and Probability; Algebra and Functions; Measurement and Geometry; Algebra 1; and Mathematical Reasoning. Each of these strands is described in detail in the following section of the Teacher Guide. For reporting purposes, the Statistics, Data Analysis, and Probability strands for grades 6 and 7 are combined. The Mathematical Reasoning test questions, which are always based on concepts in the other five strands, are reported under those strands. Thus there is no reporting category specifically for Mathematical Reasoning.

The CAHSEE is focused on mathematics constructs that are taught and assessed throughout elementary, middle, and high school. Three underlying constructs have been identified for the mathematics part of the CAHSEE: computational and procedural skills, conceptual understanding, and problem solving. Test questions on the exam cover one or more of these constructs, and CAHSEE test question writers and reviewers verify that each question measures the appropriate construct as well as the identified content standard. The constructs for each standard are given in the following pages.

Although test questions for the mathematics part of the CAHSEE do not specifically test students on mathematics vocabulary, they may require students to understand mathematical terms. It is especially important that students know the terms that appear in the language of the content standards associated with a question.

The following pages of the Teacher Guide discuss the mathematics strands and content standards included in the CAHSEE. The mathematics strands are:

- Number Sense
- Statistics, Data Analysis, and Probability
- Algebra and Functions
- Measurement and Geometry
- Algebra 1
- Mathematical Reasoning

After each strand is described, each standard in that strand is discussed in detail, and a sample released test question is provided to illustrate each standard. Also included are possible reasons that students might select the distractors, as well as an explanation of the correct answer. Teachers will find this section of the guide useful in understanding how the California content standards are assessed on the CAHSEE. A thorough understanding of the standards and the test questions associated with them will help teachers focus their instruction on the content standards and better prepare students for the exam.

The Number Sense Strand

Students' understanding of fractions, decimals, percents, and integers—and their relationship to each other and to the other disciplines of mathematics—is an essential component of their mathematics learning. CAHSEE test questions in the Number Sense strand require students to demonstrate a foundational understanding of numbers and ways they are represented.

Students will be asked to:

- solve problems with fractions, decimals, and percents
- compare and order numbers
- demonstrate an understanding of percents, including those less than 1 and greater than 100
- use ratios and proportions
- understand and meaningfully interpret large and small numbers in scientific notation
- use specific characteristics of numbers, such as multiples, factors, and primes
- use and represent integers as the basis for the comparison of quantities

Essential to success in this CAHSEE strand is the student's understanding of the mathematical operations and the ways they are related to each other. This understanding includes:

- the meaning of arithmetic operations with fractions, decimals, and integers
- the associative and commutative properties of addition and multiplication

- the distributive property of multiplication over addition
- the understanding and use of inverse relationships of addition and subtraction, multiplication, and division
- finding square roots, squaring numbers, and using the inverse relationship between them

Students also should possess computational fluency. They should be able to select appropriate methods and tools for computing with fractions and decimals; perform mental arithmetic; use algorithms for computing with fractions, decimals, and integers; use strategies for estimation and for judging the reasonableness of results; and be able to analyze and explain methods for solving problems with proportions.

The 10 California content standards covered by the CAHSEE Number Sense strand are discussed in the following pages.

Strand **Number Sense (NS)**

 Standard **NS1.1**

Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation.

 Construct **Conceptual Understanding**

The radius of the earth's orbit is 150,000,000,000 meters. What is this number in scientific notation?

A 1.5×10^{-11}

B 1.5×10^{11}

C 15×10^{10}

D 150×10^9

Scientific notation is required knowledge in science and engineering because many numbers are either so large or so close to zero that there is no other convenient way to write them. CAHSEE test questions in this standard require students to demonstrate understanding of the basic concepts of scientific notation using approximations of very large and very small numbers. Test questions may also involve the translation of approximate numbers into scientific notation, the comparison of numbers in scientific notation with either positive or negative exponents, and the understanding of the relative size of two numbers in scientific notation.

Sample Test Question

The sample question gives the radius of the earth's orbit as 150,000,000,000 meters and asks students to translate that number into scientific notation. The correct answer is choice B. Students should recognize that the place-value distance from the 1 (highest place value, 100 billion) to the decimal is 11 digits and that this value provides an appropriate representation of the equivalence as 10^{11} , also equivalent to $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$. Students also should know that expressions in scientific notation may include a multiplier, between 1 and 10, along with an exponential value of 10. While exact powers of 10 are expressed without a multiplier (e.g., 10^{11}), numbers such as 150,000,000,000 require a multiplier along with the equivalent power of 10. A typical method of finding the multiplier and the exponent is to count the number of decimal places the decimal must move to create a number between 1 and 10. In the example, the decimal point is moved 11 places to the left to get 1.5 for the multiplier and + 11 for the exponent.

Analysis of Distractors

Two of the distractors in the sample test question, C and D, represent common errors students may make in finding equivalence between standard and scientific notation: Both distractors are equivalent in number to 150,000,000,000 (and 1.5×10^{11}), but neither has a multiplier within the appropriate range (from 1 to 10). Distractor A expresses the power of 10 as $\frac{1}{10^{11}}$ (until students have a clear understanding of the magnitude of very large

and very small numbers, they are often unsure which direction the decimal point should move.)

Strand	Number Sense (NS)	Which of the following numerical expressions results in a negative number? A $(-7) + (-3)$ B $(-3) + (7)$ C $(3) + (7)$ D $(3) + (-7) + (11)$
Standard	NS1.2	
Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.		
Constructs	Procedural Skills, Conceptual Understanding	

All students should understand the basic arithmetic functions involving rational numbers in all forms and be comfortable in performing calculations with positive and negative numbers. CAHSEE test questions in this standard require students to demonstrate computational fluency with rational numbers and an understanding of the relationships between these types of numbers. These skills are fundamental to achievement of the California content standards in mathematics.

Sample Test Question

The sample question asks students to compute four combinations of values with different signs and then identify which of the sums is negative. The correct answer is choice A. Students should understand that adding rational numbers with unlike signs requires finding the difference of their absolute values and then choosing the sign of the addend with the greater absolute value. Students should also recognize that adding rational numbers with like signs requires adding the absolute values and keeping the sign, as in choice A $(-7) + (-3) = -10$.

Analysis of Distractors

The distractors present students with errors in computing with integers. Two of the distractors, B and C, have the same absolute values as the addends in the correct answer, A, with sums of 4 and 10 respectively, but they are both positive integers. Distractor D introduces a third value and has a sum of 7, but it too is a positive integer.

Strand	Number Sense (NS)	Some students attend school for 180 of the 365 days in a year. About what percentage of the year do they attend school? A 18% B 50% C 75% D 180%
Standard	NS1.3 Convert fractions to decimals and percents and use these representations in estimations, computations and applications.	
Constructs	Procedural Skills, Conceptual Understanding	

Students using mathematics in their daily lives will need to know how to convert decimals to fractions to percents with ease. The recognition of equivalent forms is essential for student fluency with numbers. CAHSEE test questions in this standard require students to demonstrate facility in finding equivalent values and representations for numbers as well as computing values using fractions, decimals, and percents. Students must also be able to use these various representations for estimating and performing computations in mathematical applications.

Sample Test Question

The sample question asks students to determine what percent 180 is of 365. The correct answer is choice B. Students should recognize that the correct value is found by dividing the part (180) by the whole (365), giving $180 \div 365 \approx 0.493$. The question also requires conversion of 0.493 to a percent by multiplying by 100 ($0.493 = 49.3\%$). Finally, students should recognize that the phrase “about what percentage” in the stem calls for a rounded percent rather than an exact value, and that $49.3\% \approx 50\%$. It is to be expected that many students will use mental estimation skills, rather than computation, to determine that 180 is about half of 365.

Analysis of Distractors

Two of the distractors, A and D, represent a misunderstanding of 180 as 18% and 180%.

Distractor C indicates incorrect computation of the decimal equivalent of $\frac{180}{365}$ or a possible guess with apparent plausibility.

Strand	Number Sense (NS)	<p>The cost of an afternoon movie ticket last year was \$4.00. This year an afternoon movie ticket costs \$5.00. What is the percent increase of the ticket from last year to this year?</p> <p>A 10%</p> <p>B 20%</p> <p>C 25%</p> <p>D 40%</p>
Standard	NS1.6	
Constructs	Procedural Skills, Conceptual Understanding	

CAHSEE test questions in this standard require students to demonstrate understanding of percent increase and decrease, which is a fundamental tool in analyzing numerical information. For example, a price change of one dollar can be very meaningful in terms of buying a loaf of bread and inconsequential in terms of buying a car. Students should understand that percent change clarifies the impact of this kind of change, and they should be able to calculate the change with facility. Standard NS1.6 will also be assessed with test questions that require students to find the percent decrease, or what quantity would result from a given percentage of increase of an original quantity.

Sample Test Question

The sample question requires students to calculate the percent of increase for a movie ticket whose price increased from \$4.00 to \$5.00. The correct answer is choice C. Students should understand that finding the percent increase or decrease of a quantity requires first finding the difference between the initial value and the final value. In the sample question, the difference is \$1.00. Then to find the percent increase, students must know to compare the difference to the initial cost by using division: $\frac{\$1.00}{\$4.00}$. Finally, the resulting decimal must be converted to its equivalent percent: $1 \div 4 = 0.25 = 25\%$.

Analysis of Distractors

The distractors represent errors resulting from a misunderstanding of the concept being tested. Distractor A results from taking the difference between the two ticket prices and converting it to a percent. Distractor B results from multiplication of the two values in the problem, \$4.00 and \$5.00. Distractor D results from converting the original ticket price to a percent.

Strand	Number Sense (NS)	Sally puts \$200.00 in a bank account. Each year the account earns 8% simple interest. How much interest will be earned in three years? A \$16.00 B \$24.00 C \$48.00 D \$160.00
Standard	NS1.7	
Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.		
Constructs	Procedural Skills, Conceptual Understanding, Problem Solving	

CAHSEE test questions in this standard require students to solve a variety of problems involving percents. Both consumers and people working in business need to understand the mathematical meaning of common business terms such as commission and profit interest computations. Solving problems of these types is one of the most important skills students need as they become adults. Understanding these concepts and their applications can mean the difference between students managing their money and other resources well, or not at all. This standard will be also be assessed with test questions that require students to find simple and compound interest, as well as discounts, markups, and commissions. A maximum of three iterations is used for questions that involve calculating compound interest. The iterations include the initial multiplication of principal by interest rate.

Sample Test Question

The sample question asks students to determine the amount of simple interest \$200 will earn in three years at the given rate. The correct answer is choice C. Students should recognize that simple interest is calculated by multiplying the principal by the annual rate and then multiplying by the time. In the sample question, the principal is \$200, the rate is 8%, and the time is 3 years. To calculate correctly, students are also required to convert 8% to its decimal equivalent ($\$200 \times .08 \times 3 = \48).

Analysis of Distractors

The distractors represent errors resulting from failure to perform one of the required steps and/or from a computation error. Distractor A represents one year's interest and results from the multiplication of the principal and the interest rate only. Distractor B represents multiplication of the rate times the number of years only. Distractor D may be attractive to students who converted the interest rate to a decimal incorrectly, multiplying \$200 by .8 and also failed to multiply by the number of years.

Strand **Number Sense (NS)**

Standard **NS2.1**

Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.

Constructs **Procedural Skills,
Conceptual
Understanding**

$$\frac{10^{-2}}{10^{-4}} =$$

A 10^{-6}

B 10^{-2}

C 10^2

D 10^8

CAHSEE test questions in this standard require students to understand the concept of negative exponents. One of the most powerful concepts in mathematics is that compact means of notation can be extended to include new concepts. Negative exponents are an example of this kind of extension.

Sample Test Question

The sample question presents a numerical calculation requiring students to demonstrate their understanding of the rule for dividing expressions involving exponents with a common base. The correct answer is choice C. Students should understand that 10^{-2} is

equivalent to $\frac{1}{10^2}$ and $\frac{1}{10^{-4}}$ is equivalent to 10^4 . Thus it is possible to represent the

problem as $10 \cdot 10 \cdot 10 \cdot 10 / 10 \cdot 10$, making the underlying concept more apparent. Once the students master the concept behind negative exponents, they understand that when dividing exponential expressions with the same base, the exponents must be subtracted,

so that $\frac{10^{-2}}{10^{-4}}$ is equivalent to $10^{(-2)-(-4)}$, which is equivalent to 10^2 .

Analysis of Distractors

The distractors represent misunderstandings of the concepts involved in the calculation.

Distractor A represents the addition of the exponents or an error in the subtraction of $(-4) - (-2)$. Distractor B shows a failure to apply the negative signs correctly in the same subtraction. Distractor D represents a failure to understand the fundamental concept, as it results from multiplying -2 by -4 .

Strand **Number Sense (NS)**

Standard **NS2.2**
Add and subtract fractions by
using factoring to find common
denominators.

Constructs **Procedural Skills,**
 Conceptual
 Understanding

Which of the following is the prime factored
form of the lowest common denominator of

$$\frac{7}{10} + \frac{8}{15} ?$$

A 5×1

B $2 \times 3 \times 5$

C $2 \times 5 \times 3 \times 5$

D 10×15

The focus of this content standard is on the students' ability to add and subtract fractions with unlike denominators that share one or more factors. Students should be able to find the prime factorization of the lowest common denominator of two whole numbers and to factor as they attempt to find a common denominator. CAHSEE test questions in this standard require students to perform addition and subtraction arithmetic using equivalent fractions with common denominators. The algorithmic approach of this standard is associated with the requirement that common denominators be determined by factoring.

Sample Test Question

The sample question requires students to find the needed common denominator for 10 and 15 using the prime factor technique for finding common denominators. The correct answer is choice B. Students should recognize that the prime factors for 10 are 2×5 and that the prime factors for 15 are 3×5 . Since the common prime factor is 5, including the additional factors of 2 and 3 gives $2 \times 3 \times 5$ as the prime factors of the least common denominator.

Analysis of Distractors

The distractors represent misunderstandings of the concept being tested. Distractor A shows 5 as a common multiple only, without the additional factors required. Distractor C incorrectly repeats 5 as a factor of the common denominator, indicating a failure to see that this value is common to both fractions. Distractor D shows the denominators multiplied by each other, as students will often use this method to find a common denominator.

Strand	Number Sense (NS)	$(3^8)^2 =$
Standard	NS2.3	A 3^4
	Multiply, divide, and simplify rational numbers by using exponent rules.	B 3^6
Constructs	Procedural Skills, Conceptual Understanding	C 3^{10}
		D 3^{16}

CAHSEE test questions in this standard require students to select the appropriate rules for operations with exponents with common bases and perform accurate computations in simplifying rational numbers. Students should understand the following rules:

- adding exponents when multiplying numbers with common bases
- subtracting exponents when dividing numbers with common bases
- multiplying exponents when raising a number to a particular power

Test questions may include those requiring multi-step operations, such as the simplification of numerators and denominators with common factors.

Sample Test Question

The sample question requires students to expand $(3^8)^2$ using the rule for multiplying exponents in parentheses $[(a^b)^c = a^{bc}]$. The correct answer is choice D. In this instance, students should simply use the rule to determine that $(3^8)^2 = 3^{16}$.

Analysis of Distractors

The distractors present the other three operations that could be performed. Distractor A represents the inappropriate operation of division, rather than multiplication. Distractor B represents the inappropriate operation of subtraction. Distractor C represents the addition of the exponents, which students might choose if they confused this calculation with one requiring multiplication of exponential expressions with the same base.

Strand **Number Sense (NS)**

Standard **NS2.4**

Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.

Constructs **Procedural Skills,
Conceptual
Understanding,
Problem Solving**

The square root of 150 is between

- A** 10 and 11.
- B** 11 and 12.
- C** 12 and 13.
- D** 13 and 14.

CAHSEE test questions in this standard require students to demonstrate a conceptual understanding of powers and roots and their inverse relationship. The idea of mathematical inverse is a key precursor for algebraic reasoning, and students should understand that roots are the inverse of raising a number or expression to a power. For students should know that $\sqrt{9^2} = 9$. Students will not be required to calculate the example, square root for a number that is not a perfect square, but students should be able to approximate the value of the square root of an integer that is not a perfect square.

Sample Test Question

The sample question asks students to find an approximation of the square root of 150, an integer that is not a perfect square. The correct answer is choice C. Students should recognize that they must first determine both the closest perfect square greater than the given integer and the closest perfect square less than the integer. This task may involve some trial and error multiplication along with the application of knowledge of squares and square roots. Since the number 150 is not a perfect square, students may recall or calculate that 144 is a perfect square and that 169 is the next perfect square. Since 144 is close to but less than 150 and 169 is close to but greater than 150, the square root of 150 must lie between those two perfect squares.

Analysis of Distractors

The distractors misplace 150 between other numbers and may be selected by students who do not understand the concept of square root or who may incorrectly calculate the square of one of the numbers.

Strand	Number Sense (NS)	If $x = 3$, what is the value of x? A -3 or 0 B -3 or 3 C 0 or 3 D -9 or 9
Standard	NS2.5	
Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.		
Constructs	Procedural Skills, Conceptual Understanding, Problem Solving	

CAHSEE test questions in this standard require students to demonstrate a conceptual understanding of absolute value and its meaning as represented on a number line. Relating absolute value to distance on the number line may help students understand the concept: Distance cannot be negative, but there will always be two numbers on the number line that are the same distance from zero. Questions may require students to find the absolute value after performing a basic computation.

Sample Test Question

The test question asks students to determine the possible values for x in a simple absolute value equation. The correct answer is choice B. Students should recognize that since the absolute value of a number is the distance on a number line from that number to zero in either direction, all absolute values are positive numbers: $|x| = x$ and $|-x| = x$. In the test question, if $|x| = 3$, then $x = -3$ or $x = 3$, since $|3| = 3$ and $|-3| = 3$.

Analysis of Distractors

The distractors represent misunderstandings of the concept and notation for absolute value. Distractors A and C incorrectly equate the absolute value of 3 with 0 and offer either a negative or a positive value of 3. Distractor D correctly provides both a negative and a positive value but inappropriately associates the absolute value of 3 with the square of 3.

The Statistics, Data Analysis, and Probability Strand

To demonstrate knowledge and skills in the Statistics, Data Analysis, and Probability strand, students must understand the fundamental concepts involved in data collection, display, and analysis. Students will be asked to determine ways to collect, organize, and display relevant data to answer questions, formulate questions that can be addressed with data, select and use appropriate statistical methods to analyze data, and develop and evaluate inferences and predictions that are based on data. In addition, students are required to understand and apply the basic concepts of probability.

Specifically, the standards in the Statistics, Data Analysis and Probability strand include the following knowledge and skills:

- finding measures of central tendency to characterize data
- interpreting and evaluating conclusions based on data
- organizing and representing possible outcomes for events and expressing theoretical probabilities
- representing probabilities as ratios, proportions, and percents
- understanding the numerical continuum of probability between impossibility (0) and absolute certainty (1)
- recognizing the difference between independent and dependent events
- displaying data appropriately, including both one- and two-variable data sets
- understanding and computing quartiles

The eight California content standards covered by the CAHSEE Statistics, Data Analysis, and Probability strand are discussed in the following pages.

Strand	Statistics, Data Analysis, and Probability (P)	Rico's first three test scores in biology were 65, 90, and 73. What was his mean score? A 65 B 73 C 76 D 90
Standard	6P1.1	
	Compute the mean, median, and mode of data sets.	
Constructs	Procedural Skills, Conceptual Understanding	

One of the major objectives of the Statistics, Data Analysis, and Probability strand is to give students tools to help them understand the uses and misuses of statistics. This CAHSEE content standard has three components: computation of the mean, computation of the median, and recognition of the mode of data sets. Statistical measures of central tendency represent important methods for summarizing and comparing single-variable data sets. Students should understand the significance of each as a measure of central tendency as well as the differences among these measures. While most students are familiar with calculating the mean (average) from dealing with grades, they should also be able to find the median of a data set and identify the mode. For this standard, students will not be asked to find the median of an even number of values.

Students should know that

- the median is the middle score of an ordered set of numbers, where half the scores are greater than the median and half are less
- the mode is the number which appears more frequently
- the mean is most affected by extreme values

Sample Test Question

The sample question gives the data set (65, 90, 73) and asks students to compute the mean. The correct answer is choice C. Students should recognize that they should compute the mean by first finding the sum ($65 + 90 + 73 = 228$) and then dividing by 3 ($228 \div 3 = 76$).

Analysis of Distractors

The distractors represent conceptual misunderstandings about measures of central tendency. Distractor A is the minimum of the data set; distractor B is the median of the data set; and distractor D is the maximum of the data set.

Strand **Statistics, Data
Analysis, and
Probability (P)**

Standard **6P2.5**
**Identify claims based on
statistical data and, in simple
cases, evaluate the validity of the
claims.**

Constructs **Conceptual
Understanding,
Problem Solving**

**Three-fourths of the 36 members of a club
attended a meeting. Ten of those attending
the meeting were female. Which one of the
following questions can be answered with
the information given?**

- A** How many males are in the club?
- B** How many females are in the club?
- C** How many male members of the club
attended the meeting?
- D** How many female members of the club
did not attend the meeting?

This content standard has two components: identifying claims made on the basis of statistical data and evaluating the validity of the claims based on statistical data. Because students should be able to understand statistical claims as well as they understand purely verbal arguments for or against a position, students should develop skills to evaluate the quality of data and conclusions based on data. CAHSEE test questions for this standard may ask students to identify a valid claim based on data or to recognize a question for which the data could be used to provide an answer.

Sample Test Question

The sample test question presents data about members of a club: Three fourths of the members, or 27 of 36, attended a meeting, and 10 were female. The answer choices represent four possible questions that these data might be used to answer. The correct answer is choice C. Students should recognize that the information given is mostly about those attending the meeting, and little is known about the members who did not attend. Thus C is the correct response because the number of male members attending the meeting can be calculated by subtracting the number of females in attendance from the total ($27 - 10 = 17$).

Analysis of Distractors

The questions presented in distractors A and D cannot be answered, based on the data, because it is not known how many of the members who did not attend (one fourth of 36) are male or female. Although distractor B might look like an attractive choice because, like the stem of the question, it deals with females, it also cannot be answered with the given information: Although it is known how many females attended the meeting, there is no information about the number of females who were not at the meeting.

Strand **Statistics, Data Analysis, and Probability (P)**

Standard **6P3.1**
Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome.

Constructs **Conceptual Understanding, Problem Solving**

To get home from work, Curtis must get on one of the three highways that leave the city. He then has a choice of four different roads that lead to his house. In the diagram below, each letter represents a highway, and each number represents a road.

		Highway		
		A	B	C
Road	1	A 1	B 1	C 1
	2	A 2	B 2	C 2
	3	A 3	B 3	C 3
	4	A 4	B 4	C 4

If Curtis randomly chooses a route to travel home, what is the probability that he will travel Highway B and Road 4?

- A $\frac{1}{16}$
- B $\frac{1}{12}$
- C $\frac{1}{4}$
- D $\frac{1}{3}$

Organizing structures such as sample spaces, diagrams, and tables are useful for the representations of probabilities, and the ability to create a structured representation of a complex situation is an important reasoning tool. To demonstrate achievement on this standard, students must recognize appropriate and correct representations of events. From the correct representation, they must derive an understanding of the relationship between the frequency of the outcome and its numerical expression. They should be able to determine a theoretical probability of any particular outcome based on a correct representation.

Sample Test Question

The sample question includes a diagram that represents combinations of route choices (3 highways, 4 roads) and asks students to determine the probability of an individual randomly taking a given route. The correct answer is choice B. Students should recognize that if there are n possible outcomes for an independent event and r possible outcomes for another independent event, there are nr outcomes for the two events together. In other words, if there are n ways to do one thing and r ways to do another thing, there are nr ways to do the two things together. Thus three highways \times four roads $= 3 \times 4 = 12$ routes, and the probability of selecting any individual route $= \frac{1}{12}$.

Analysis of Distractors

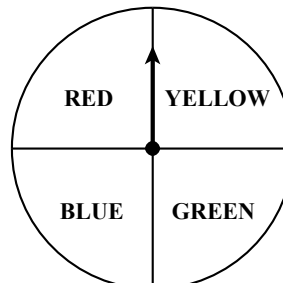
The distractors represent misunderstandings of the ways to determine probability and/or failure to complete the steps in the problem. Distractor A represents an inappropriate squaring of the probability of taking one road. Distractor C represents the probability of taking any one road, and distractor D represents the probability of taking any one highway.

Strand **Statistics, Data
Analysis, and
Probability (P)**

Standard **6P3.3**
**Represent probabilities as ratios,
proportions, decimals between 0
and 1, and percentages between 0
and 100, and verify that the
probabilities computed are
reasonable; know that if P is the
probability of an event, 1-P is the
probability of an event not
occurring.**

Constructs **Conceptual
Understanding,
Problem Solving**

**What is the probability that the spinner
will NOT stop on red if it is spun one
time?**



- A $\frac{1}{4}$
- B $\frac{1}{3}$
- C $\frac{3}{4}$
- D $\frac{4}{3}$

All students should understand that mathematical probability is used to predict what might happen in the future and that probabilities are ratios determined by considering the likely results or outcomes of events. CAHSEE test questions for this standard cover all of the components of the standard. Students are expected to:

- know that probabilities are ratios that can be expressed as fractions, decimals, or percentages
- compute the probability of a described event
- verify the reasonableness of a computed probability
- compute the probability that an event will not occur

Sample Test Question

The sample question asks students to determine the probability that an event will not occur—that a spinner will not land on one quadrant of a circle. The correct answer is choice C. Students should recognize that because the four regions covered by the spinner have equal areas, the probability of the spinner stopping on any one region is $\frac{1}{4}$.

Therefore the probability that it will **not** stop on a given region is $1 - \frac{1}{4}$, or $\frac{3}{4}$.

Analysis of Distractors

The distractors represent misconceptions about the probability of an event not occurring. Distractor A gives the probability that the event will occur; this value has not been subtracted from 1. Distractor B represents a misconception that only three quadrants should be used to calculate the probability and gives the probability of the spinner landing on one of the three. Distractor D represents either of two possibilities: using a fraction to represent the total number of quadrants divided by three quadrants (all but red) or obtaining the correct answer but then inverting it.

Strand **Statistics, Data
Analysis, and
Probability (P)**

Standard **6P3.5**
**Understand the difference
between independent and
dependent events.**

Construct **Conceptual
Understanding**

A bag contained four green balls, three red balls, and two purple balls. Jason removed one purple ball from the bag and did not put the ball back in the bag. He then randomly removed another ball from the bag. What is the probability that the second ball Jason removed was purple?

A $\frac{1}{36}$

B $\frac{1}{9}$

C $\frac{1}{8}$

D $\frac{2}{9}$

Discerning the difference between dependent and independent events is important in evaluating probabilistic outcomes. CAHSEE test questions in this content standard require students to understand that events are independent of each other if the occurrence or non-occurrence of one event does not affect the probability of the occurrence or non-occurrence of another event. Similarly, students must recognize that events are dependent if the occurrence or non-occurrence of one event affects the probability of the occurrence or non-occurrence of another event. Computation may be required to determine the result of the independent or dependent events.

Sample Test Question

The test question asks students to demonstrate understanding of the probability of the occurrence of a dependent event. The correct answer is choice C. Students should determine that initially the bag contains 4 green, 3 red, and 2 purple balls, for a total of 9 balls, and that when 1 purple ball is removed and not replaced, the bag contains a total of 8 balls. Since there are now 8 balls with only 1 being purple, the probability of randomly choosing the purple ball is $\frac{1}{8}$.

Analysis of Distractors

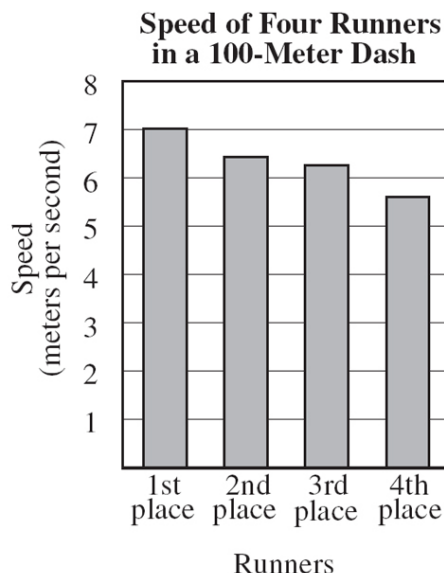
The distractors offer misunderstandings of the underlying concepts in the problem.

Distractor A results from the inappropriate multiplication of $\frac{2}{9} \times \frac{1}{8}$, which is the probability of randomly choosing both purple balls. Distractor B results from computing the probability of choosing 1 purple ball from 9 balls. Distractor D results from the addition of two probabilities for independent events: $\frac{1}{9} + \frac{1}{9}$.

Strand **Statistics, Data Analysis, and Probability (P)**

Standard **7P1.1**
Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**



Based on the bar graph shown above, which of the following conclusions is true?

- A** Everyone ran faster than 6 meters per second.
- B** The best possible rate for the 100-meter dash is 5 meters per second.
- C** The first-place runner was four times as fast as the fourth-place runner.
- D** The second-place and third-place runners were closest in time to one another.

Large data sets are difficult to grasp mentally without an accessible visual representation. CAHSEE test questions in this standard require students to recognize and interpret various forms of display and to compare two sets of data displayed the same way. The forms of display for single-variable data sets assessed on the CAHSEE include bar graphs, line graphs, pictograms, stem and leaf plots, box and whisker plots, and circle graphs. Students should be able to determine subsets of the following: median and mode, minimum and maximum, upper and lower quartiles, and a comparison of percentages in a whole. CAHSEE test questions for this standard may also require students to select an appropriate type of data display.

Sample Test Question

The sample question presents a bar graph that shows the average speed, in meters per second (m/s), of four runners over a 100-meter distance and asks students to identify an accurate conclusion supported by the data. The correct answer is choice D. Students should use the graph to determine the speed of each runner and then evaluate and

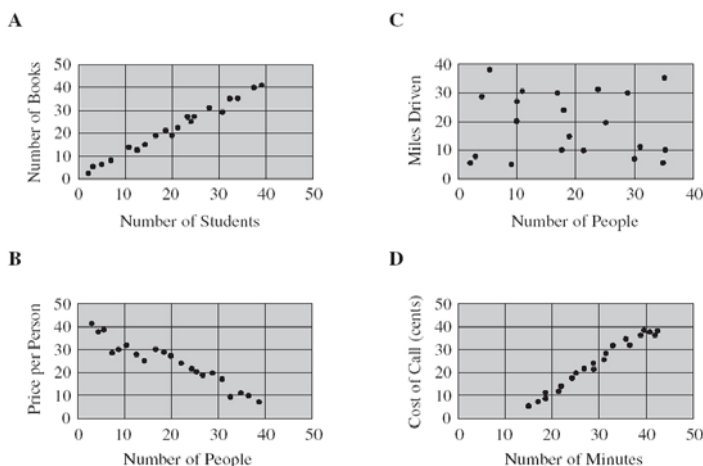
compare the four answer choices. From the graph, the first-place runner's speed was approximately 7.1 m/s; the second place runner's speed was approximately 6.5 m/s; the third place runner's speed was approximately 6.3 m/s; and the fourth place runner's speed was approximately 5.6 m/s. This analysis shows that the second place and third place runners' times were closest together.

Analysis of Distractors

Students should recognize that distractor A is incorrect because the fourth place runner ran at approximately 5.6 m/s; distractor B is incorrect because all four runners ran faster than 5 m/s; and distractor C is incorrect because 7.1 m/s is not four times faster than 5.6 m/s.

Strand	Statistics, Data Analysis, and Probability (P)
Standard	7P1.2
	Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).
Constructs	Conceptual Understanding Problem Solving

Which scatter plot shows a negative correlation?



The identification of patterns and relationships, including clustering and trends, as well as the concept of correlation (positive, negative, or none) are significant aspects of using data. Students should understand correlation as a measure of the relationship between two variables, with negative correlation as the association of an increase in the value of one variable with a decrease in the corresponding value of the second variable. Students should also recognize that positive correlation is the association of an increase in the value of one variable with an increase in the corresponding value of the second variable.

CAHSEE test questions for this standard address the following components of the standard:

- representing two variables on a scatterplot
- determining the distribution of the variables
- recognizing the apparent relationships between the two variables represented

Sample Test Question

The sample item requires students to identify the graph that illustrates a negative correlation. The correct answer is choice B: As the price per person increases, the number of people decreases, indicating a negative correlation.

Analysis of Distractors

The distractors are plots that show either a positive correlation or no correlation. Distractor A represents a positive correlation: As the number of books increases, the number of students increases. Distractor C indicates no correlation: As miles driven increases, the number of people both increases and decreases over the range of miles driven. Distractor D indicates a positive correlation: As the cost of one call increases, the number of minutes increases (at or greater than values of 15).

Strand	Statistics, Data Analysis, and Probability (P)	Joel's scores on eight English quizzes are 11, 12, 14, 15, 17, 18, 20, and 21. What is the upper quartile value of the scores? A 18 B 19 C 20 D 21
Standard	7P1.3	
	Understand the meaning of, and be able to compute the minimum, the lower quartile [value], the median, the upper quartile [value], and the maximum of a data set.	
Constructs	Procedural Skills, Conceptual Understanding	

Certain data sets require the interpretation of one value in relation to the entire data set. If the values of a data set are arranged from lowest to highest, the minimum, lower quartile, maximum and upper quartile divide the data into four groups that are approximately the same size. This provides useful and meaningful representation of data. CAHSEE test questions for this standard require students to compute and identify, for various lists of data, the following specific data characteristics:

- minimum
- lower quartile
- median
- upper quartile
- maximum

Students should understand that if the values are in random order, they must first be ordered before calculating or identifying a specific value. Students must also be able to interpret these values from graphical representations such as stem-and-leaf plots or bar graphs.

Sample Test Question

The sample question provides a data set and asks students to determine the upper quartile value. The correct answer is choice B. Students should understand that the upper quartile is the median of the top half of the values in the data set. Since there are 8 values in all, there are 4 in the upper half: 17, 18, 20, and 21. Therefore the upper quartile is the mean of the 2 middle values (18 and 20).

Analysis of Distractors

The distractors represent misunderstandings of the concept of quartiles. Distractors A and C may be attractive to students who assume that a median or quartile value must be present within the data set. Distractor D, the maximum, will be attractive to students who incorrectly associate the upper quartile with maximum value.

The Algebra and Functions Strand

The Algebra and Functions strand is most closely associated with the representation of quantitative relationships as a style of mathematical thinking for formalizing patterns, functions, and generalizations. As students increase their mathematical knowledge and skills, they work frequently with algebraic symbols, expressions with variables, and graphical representations. It is essential that students develop an understanding of several different meanings and uses of variables through multiple representations. Everyday experiences with linear functions should aid in the development of the concepts of proportionality and the ability to discriminate between linear and nonlinear functions. Students must also learn to recognize and generate equivalent expressions, solve linear equations, and effectively use formulas.

To demonstrate achievement in this strand, students will be asked to:

- work with patterns and relationships
- represent, analyze, and generalize a variety of patterns with table, graphs, and symbolic rules
- compare different forms of representations
- identify functions
- use algebraic expressions
- solve linear equations

The use of mathematical models to represent and understand quantitative relationships is developed by modeling and solving contextualized problems. The analysis of change in various contexts involves tools such as graphs to analyze the nature of changes in quantities in linear relationships.

The 10 California Content Standards covered by the CAHSEE grade 7 Algebra and Functions strand are discussed in the following pages.

Strand	Algebra and Functions (AF)	<p>Which of the following inequalities represents the statement, “A number, x, decreased by 13 is less than or equal to 39”?</p> <p>A $13 - x \geq 39$</p> <p>B $13 - x \leq 39$</p> <p>C $x - 13 \leq 39$</p> <p>D $x - 13 < 39$</p>
Standard	AF1.1 Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).	
Constructs	Procedural Skills, Conceptual Understanding, Problem Solving	

Translating verbal descriptions into mathematical expressions is essential in solving real-world problems. CAHSEE test questions in this standard require students to translate between verbal descriptions and mathematical equivalents. Students should be able to use variables and appropriate operations to write or identify an expression, an equation, a system of equations or an inequality to solve a problem. Test questions may also ask students to set up an appropriate equation.

Sample Test Question

The sample question requires students to translate a verbal description of an inequality into a mathematical expression. The correct answer is choice C. Students should recognize that “a number, x , decreased by 13” is represented as $x - 13$ and that “less than or equal to 39” is represented by ≤ 39 . Putting both parts of the statement together, $x - 13 \leq 39$.

Analysis of Distractors

The distractors offer expressions that use the same values found in the problem but represent verbal expressions not given in the stem. Distractor A represents “13 decreased by a number, x ,” and also “greater than or equal to 39.” Distractor B uses the correct inequality notation but, like distractor A, represents “13 decreased by a number, x .” Distractor D presents the appropriate expression for “a number, x , decreased by 13” but represents “less than 39,” rather than “less than or equal to 39.”

Strand	Algebra and Functions (AF)	If $h = 3$ and $k = 4$, then $\frac{hk + 4}{2} - 2 =$ A 6 B 7 C 8 D 10
Standard	AF1.2	
	Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$	
Constructs	Procedural Skills, Conceptual Understanding	

CAHSEE test questions for this standard require students to select and use the correct order of arithmetic operations in evaluating expressions (parentheses, exponents, multiplication, division, addition, subtraction). Students may also be required to evaluate expressions that include the distributive property and other basic properties of real numbers.

Sample Test Question

The sample question asks students to evaluate an expression using the correct order of operations. The correct answer is choice A. Students should first multiply h by k , add 4, divide by 2, and then subtract 2: $\frac{(3)(4)+4}{2} - 2 = \frac{12+4}{2} - 2 = \frac{16}{2} - 2 = 8 - 2 = 6$.

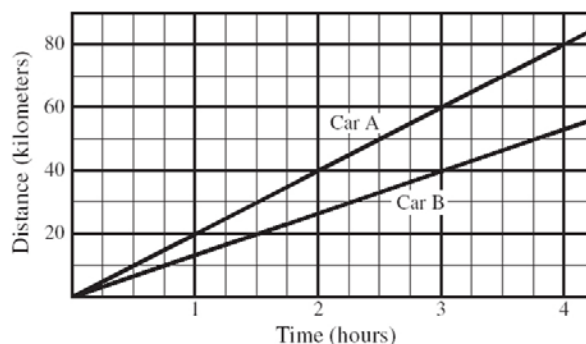
Analysis of Distractors

The distractors offer solutions that result from performing operations in improper order. Distractor B is obtained by dividing by 2 before evaluating the numerator. Distractor C is obtained by dividing the product of 3 and 4 by 2 before evaluating the numerator. Distractor D is obtained by adding 4 before multiplying 3 by 4.

Strand **Algebra and Functions (AF)**

Standard **AF1.5**
Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

Constructs **Conceptual Understanding, Problem Solving**



After three hours of travel, Car A is about how many kilometers ahead of Car B?

- A 2
- B 10
- C 20
- D 25

CAHSEE test questions for this standard focus on either of its two main components. The first component requires the selection and execution of a graph that accurately and appropriately represents a quantitative relationship. The second component requires the interpretation and/or alternate representation of information presented in graphical form.

Sample Test Question

The sample question presents a graph showing distance traveled over time for two cars, A and B, and asks for a specific interpretation of the information shown in the graph. The correct answer is choice C. Students should recognize that distance, in kilometers, is recorded on the y-axis, and time, in hours, is recorded on the x-axis. Three hours on the time scale corresponds to 60 kilometers for Car A, and three hours corresponds to 40 kilometers for Car B. The number of kilometers that Car A is ahead of Car B after 3 hours is represented by the difference between the distance traveled by Car A and the distance traveled by Car B in the same time (60 kilometers – 40 kilometers = 20 kilometers).

Analysis of Distractors

The distractors represent misreadings of the graph. Distractor A gives the number of hours that Car A has traveled when it has gone 40 kilometers, the distance Car B traveled in 3 hours. Distractor B represents an error in reading the scale of the graph, assuming that the increments have a value of 10. Distractor D represents use of the approximate difference in distance at 4 hours, rather than 3.

Strand **Algebra and
Functions (AF)**

Standard **AF2.1**
**Interpret positive whole-number
powers as repeated multiplication
and negative whole-number
powers as repeated division or
multiplication by the
multiplicative inverse. Simplify
and evaluate expressions that
include exponents.**

Constructs **Procedural Skills,
Conceptual
Understanding**

$$x^3 y^3 =$$

A $9xy$

B $(xy)^6$

C $3xy$

D $xxxyyy$

This standard has four main components: the concept of positive whole-number powers as repeated multiplication, the concept of negative whole-number powers as repeated division, multiplication by the multiplicative inverse, and simplification and evaluation of expressions that include exponents. The first two components of this standard are assessed in the related Number Sense standards 2.1 and 2.3. CAHSEE test questions that assess student achievement in this standard may require students to evaluate monomial expressions. Other questions for this standard may require students to demonstrate an understanding of the multiplicative inverse.

Sample Test Question

The sample question asks students to interpret as repeated multiplication the algebraic expression for x raised to the third power multiplied by y raised to the third power. The correct answer is choice D. Students should recognize that $x^3 = x \cdot x \cdot x$ and that $y^3 = y \cdot y \cdot y$, so that $x^3 y^3 = x \cdot x \cdot x \cdot y \cdot y \cdot y = xxxyyy$.

Analysis of Distractors

The distractors represent misunderstandings of the meaning of the exponents and/or the appropriate operation. Distractor A multiplies the exponents and uses the product as a coefficient. Distractor B adds the exponents, as would be appropriate for like, rather than unlike, variables. In distractor C, the value of the exponents has been moved to serve as a coefficient.

Strand **Algebra and
Functions (AF)**

Standard **AF2.2**
**Multiply and divide monomials;
 extend the process of taking
 powers and extracting roots to
 monomials when the latter results
 in a monomial with an integer
 exponent.**

Construct **Conceptual
Understanding**

Simplify the expression shown below.

$$(6a^4bc)(7ab^3c)$$

- A $13a^4b^3c$
- B $13a^5b^4c^2$
- C $42a^4b^3c$
- D $42a^5b^4c^2$

CAHSEE test questions in this standard require students to multiply and divide monomials of the form Ax^n and to expand powers and find roots to monomials of the same form whose roots have integer exponents. Items may also include combinations of multiplying and dividing monomials. Students should be comfortable with the rules for multiplying and dividing exponential expressions with the same base.

Sample Test Question

The sample question requires students to demonstrate knowledge of the rules for multiplying monomials, multiplying the numeric values and adding the exponents. The correct answer is choice D, as $6 \times 7 = 42$ and the product of a^4 and a^1 is a^5 ; the product of b^1 and b^3 is b^4 ; and the product of c^1 and c^1 is c^2 .

Analysis of Distractors

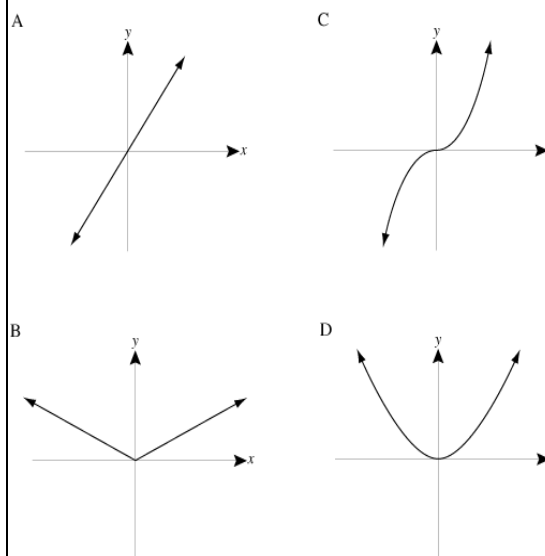
The distractors represent errors in multiplying the numeric values and/or the exponents. Distractor A represents the incorrect operation of adding, rather than multiplying, 6 and 7 and also the failure to use a as a^1 , b as b^1 , and c as c^1 when adding the values of the exponents. Distractor B represents the incorrect value obtained by adding 6 and 7 combined with the correct values for the exponents. Distractor C represents the correct multiplication of the numeric values but, like distractor A, has the incorrect multiplication of the exponents a , b , and c .

Strand **Algebra and Functions (AF)**

Standard **AF3.1**
Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.

Construct **Conceptual Understanding**

Which of the following could be the graph of $y = x^3$?



CAHSEE test questions in this standard require knowledge of graphing functions, as demonstrated by selecting the appropriate graph of a given function or by selecting the appropriate function for a given graph. Some questions may also require knowledge of function graphing to solve problems. As part of their foundational understanding of functions, students should be able to predict the shape of a graph based on the characteristics of the given function (e.g., linear, quadratic).

Sample Test Question

The sample question asks students to identify which graph could represent the function $y = x^3$. The correct answer is choice C. Students should understand the basic concepts underlying the problem—that cubic functions are nonlinear and that negative values for x correspond to negative values for y and positive values for x correspond to positive values for y .

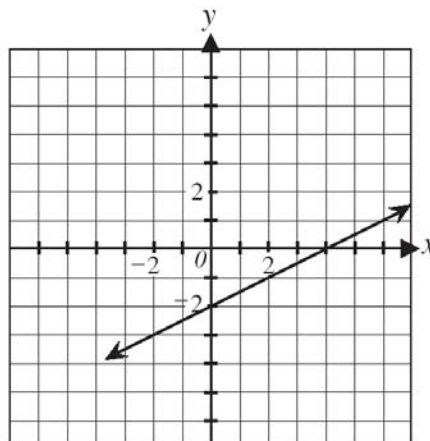
Analysis of Distractors

Distractors A and B are graphs of linear, rather than nonlinear, functions, and distractor D represents a quadratic function in which negative values for x correspond to positive values for y .

Strand **Algebra and Functions (AF)**

Standard **AF3.3**
Graph linear functions, noting that the vertical change (change in y -value) per unit of horizontal change (change in x -value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.

Constructs **Procedural Skills, Conceptual Understanding**



What is the slope of the line shown in the graph above?

- A -2
- B $-\frac{1}{2}$
- C $\frac{1}{2}$
- D 2

Students should understand that linear functions can model many real-world phenomena and that the rate of change in a function is shown by the slope of the graph of the function. A conceptual understanding of slope can be a key element in students' development of proportional reasoning skills. CAHSEE test questions for this standard may focus on either of its two main components. The first component is graphing linear functions on the xy coordinate system. The second is the identification of the slope in quantitative terms from a given linear function or the selection of a given slope from a numerical value, from a line shown on a graph, or from two pairs of coordinate points.

Sample Test Question

The sample question shows a graph of a linear function and asks students to determine the slope represented by a line that crosses the y -axis at the point $(0, -2)$ and the x -axis at the point $(4, 0)$. The correct answer is choice C. Students should understand slope as the change in y divided by the change in x and/or as the ratio "rise over run." In this problem, the change in the y -value is obtained by subtracting -2 from 0 , and the change in the x -value is obtained by subtracting 0 from 4 , and thus

$$\frac{\text{Change in } y}{\text{Change in } x} = \frac{0 - (-2)}{4 - 0} = \frac{2}{4} = \frac{1}{2}.$$

Analysis of Distractors

Distractor A is simply the value of the y -intercept and represents a misunderstanding of the concept of slope. Distractor B represents an error in the subtraction of $0 - (-2)$ or $4 - 0$. Distractor D represents the error of dividing the change in x by the change in y .

Strand Algebra and Functions (AF)

Standard AF3.4

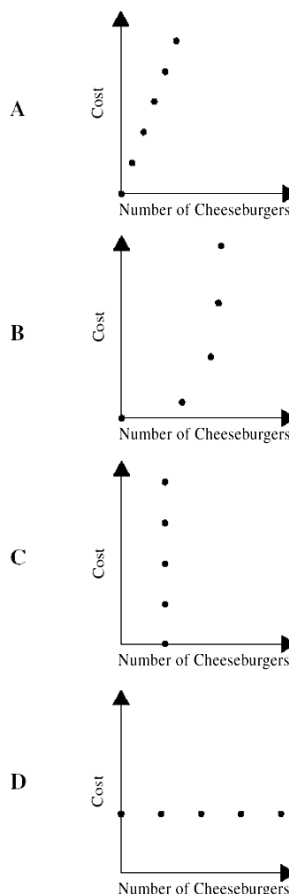
Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of a line equals the [ratio of the] quantities.

Construct Conceptual Understanding

10. Best Burger sells cheeseburgers for \$1.75 each. Part of a table representing the number of cheeseburgers purchased and their cost is shown below.

Number Purchased	Cost(\$)
0	0
1	1.75
2	3.50
3	5.25
4	7.00

Which of the following is a portion of the graph of the data in the table?



Graphing direct variation is a powerful way to comprehend and express proportional reasoning. CAHSEE test questions within this standard focus on either of its two main components, both of which require students to understand the relationship between the graphical presentation of data and the symbolic representation of data. The first component involves the identification of the correct graph. The second component involves the determination of the slope of a direct variation and the interpretation of the meaning of the slope as a constant ratio between the two quantities in the variation.

Sample Test Question

The sample question presents a table of values showing the number of cheeseburgers purchased and the corresponding cost for each additional cheeseburger. Students are asked to select the appropriate graph to represent the data. The correct answer is choice A. Students must understand the relationship between the number of cheeseburgers on the x -axis and the corresponding cost for each additional cheeseburger on the y -axis. The table shows that each additional cheeseburger purchased increases the total cost by \$1.75. In other words, students should recognize that the relationship is linear: As the value on the x -axis increases by 1, the value on the y -axis increases correspondingly by \$1.75.

Analysis of Distractors

The distractors are graphs that incorrectly represent the function in the table. Distractor B shows a nonlinear relationship between number and cost. Cost increases exponentially, so that following purchase of cheeseburger number three, it has risen higher than the \$7.00 indicated in the table. Distractor C represents a constantly increasing cost without the purchase of additional cheeseburgers after the first one. Distractor D represents the purchase of additional cheeseburgers without any cost increase beyond the cost of the first cheeseburger.

Strand **Algebra and
Functions (AF)**

Standard **AF4.1**
**Solve two-step linear equations
and inequalities in one variable
over the rational numbers,
interpret the solution or solutions
in the context from which they
arose, and verify the
reasonableness of the results.**

Constructs **Procedural Skills,
Conceptual
Understanding,
Problem Solving**

**In the inequality $2x + \$10,000 \geq \$70,000$,
 x represents the salary of an employee in a
school district. Which phrase most
accurately describes the employee's salary?**

- A** At least \$30,000
- B** At most \$30,000
- C** Less than \$30,000
- D** More than \$30,000

CAHSEE test questions for this standard may focus on any of its components, including solving two-step linear equations, solving two-step inequalities, interpreting the solutions of equations or inequalities, and judging the reasonableness of the solutions or equations or inequalities.

Sample Test Question

The sample test question presents an inequality that represents an employee's salary and asks students to interpret the solution of the inequality in terms of this context. The correct answer is choice A. Students should recognize that the inequality has the solution $x \geq \$30,000$ and that the correct way to state this inequality is "at least \$30,000."

Analysis of Distractors

The distractors offer incorrect solutions for the inequality and/or ways to express the mathematical notation. Distractor B represents $x \leq \$30,000$, which is stated as "at most \$30,000." Distractor C represents $x < \$30,000$, or "less than \$30,000." Distractor D represents $x > \$30,000$, or "more than \$30,000."

Strand	Algebra and Functions (AF)	<p>The diameter of a tree trunk varies directly with the age of the tree. A 45-year-old tree has a trunk diameter of 18 inches. What is the age of a tree that has a trunk diameter of 20 inches?</p> <p>A 47 years</p> <p>B 50 years</p> <p>C 63 years</p> <p>D 90 years</p>
Standard	AF4.2	
	Solve multi-step problems involving rate, average speed, distance, and time or a direct variation.	
Constructs	Procedural Skills, Conceptual Understanding, Problem Solving	

Problem solving is a significant higher-order thinking skill that enables students to apply their mathematical knowledge to real-world situations. CAHSEE test questions for this standard may require students either to solve a specific problem or to determine the equation that should be used to solve the problem. Test questions may also require students to understand the concept of direct variation and to recognize that direct variation may also be expressed as a linear function. This standard is closely related to Algebra 1 standard 5.0, which requires students to solve a variety of problems such as rate, work, and percent mixture using algebraic methods.

Sample Test Question

The sample question provides a ratio between the age of a tree and the diameter of its trunk and asks students to determine the age of a second tree, given its diameter. The correct answer is choice B. Students may choose to use a variety of appropriate solutions that rely on the correspondence between the age of the tree and its diameter. One method is to develop the equation representing the direct variation between the ages of the trees and their corresponding diameters: $\frac{45}{18} = \frac{x}{20}$ and solve for x .

Analysis of Distractors

The distractors offer solutions that use the values of the problem incorrectly and thus reflect a misunderstanding of the concept of direct variation. Distractor A represents the error of adding the value of the difference between the two diameters ($20 - 18 = 2$) to the age of the first tree in order to obtain the age of the second tree. Distractor C represents the addition of the two values associated with the first tree ($45 + 18 = 63$). Distractor D represents the product of the age of the first tree (45) and the difference between the two diameters (2).

The Measurement and Geometry Strand

As students relate their experiences from earlier classroom instruction in measurement and geometry to situations in their everyday lives, their knowledge and ability to apply this knowledge increase in depth and sophistication. To demonstrate understanding in this CAHSEE strand, students must be able to select and use appropriate units, estimate and calculate measurements for the length, area, and volume of geometric figures, understand scaling in scale drawings and how changes in linear dimension affect area and volume, and solve problems involving dimensional analysis and conversion from one unit to another.

To demonstrate achievement of the knowledge and skills in the measurement component of the strand, students should be able to use both metric and customary units of measurement for the following:

- determining the relationship between different units within the same system and converting from one unit to another within and between measurement systems
- using scale drawings and models to determine measurements of the original
- solving problems involving dimensional analysis for rates and other compound units
- relating the effect of changing the choice of a linear unit on the related square and cubic units for area and volume, respectively

The geometry component of this strand includes computing the perimeter, area, and volume of the most common 2- and 3-dimensional figures, using these common figures to estimate or compute the area of more complex objects.

To demonstrate acquisition of the knowledge and skills in the geometry component of the strand, students should be able to:

- describe, classify, and understand relationships between length, area, and volume among types of 2- and 3-dimensional objects
- use coordinate geometry to represent and examine the properties of figures and their images under translation and reflection
- understand and use the Pythagorean theorem
- recognize and demonstrate understanding of congruence in terms of the sides and angles of 2-dimensional figures

When CAHSEE items require students to use formulas, the formulas are provided within parentheses in the stem. All formulas the students may use, including the estimated value of π , will be provided *except* for the following:

- Perimeter of a polygon and the circumference of a circle
- Area of a triangle or parallelogram (including rectangles)
- Volume of a rectangular prism

The formula for finding the area of a nontraditional figure such as a rhombus will be provided.

The 10 California Content Standards covered by the CAHSEE Measurement and Geometry strand are discussed in the following pages.

Strand	Measurement and Geometry (MG)	Juanita exercised for one hour. How many seconds did Juanita exercise? A 60 B 120 C 360 D 3,600
Standard	MG1.1	
	Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).	
Constructs	Procedural Skills, Conceptual Understanding	

Comparing units and computing the effect of changing units are essential skills for acquiring knowledge in mathematics and science disciplines. CAHSEE test questions for this standard require students to convert between two units of measurement within the same system or between two different systems of measurement. Conversion formulas are provided for test questions that require the student to convert between less commonly utilized units (such as pints to gallons), systems of measurement, square units, and cubic units.

Sample Test Question

The sample question requires students to convert common units of time. The correct answer is choice D. Students must compute the number of seconds in 1 hour by multiplying the 60 seconds in 1 minute by the 60 minutes in 1 hour, or 60×60 , or 3600 seconds.

Analysis of Distractors

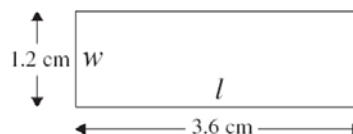
The distractors represent errors in performing the conversions. Distractor A gives the number of seconds in 1 minute or the number of minutes in 1 hour and thus represents the failure to perform the second step of the problem. Distractor B represents adding, rather than multiplying, the number of seconds in a minute and the number of minutes in an hour. Distractor C represents a computation error in multiplying 60 by 60.

Strand **Measurement and Geometry (MG)**

Standard **MG1.2**
Construct and read drawings and models made to scale.

Constructs **Conceptual Understanding, Problem Solving**

The actual width (w) of a rectangle is 18 centimeters (cm). Use the scale drawing of the rectangle to find the actual length (l).



- A 6 cm
- B 24 cm
- C 36 cm
- D 54 cm

The ability to move back and forth between a scale drawing or model and a real object is essential for understanding representation and well as proportional reasoning. Test questions on the CAHSEE focus on the second component of this standard, reading scale drawings and models. Students will be asked to read and interpret drawings and models made to scale. Students may also be asked to apply given measurements to determine the scale of a figure.

Sample Test Question

The sample test question presents a scale drawing of a rectangle with width 1.2 cm and length 3.6 cm and gives the actual value of the width as 18 cm. The correct answer is choice D. Students must recognize that the relationship between the width of the scale drawing and the actual width of the rectangle is the same as that between the length of the scale drawing and the actual length. Students may use a variety of approaches to solve the problem, including setting up a proportion similar to $\frac{1.2}{18} = \frac{3.6}{l}$ and solving for l .

Analysis of Distractors

The distractors represent errors in using the values presented in the stem. Distractor A results from use of an incorrect relationship between the parts of the proportion: $(18 \div 3.6) \times 1.2 = 6$. Distractor B results from subtracting the width from the length and multiplying by 10. Distractor C results from using 3.6 as twice, rather than three times, 1.2 and then multiplying by 2.

Strand	Measurement and Geometry (MG)	Sixty miles per hour is the same rate as which of the following? A 1 mile per minute B 1 mile per second C 6 miles per minute D 360 miles per second
Standard	MG1.3 Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.	
Constructs	Procedural Skills, Conceptual Understanding, Problem Solving	

Dimensional analysis is a critical skill in physical science, engineering, and the social sciences. Students should be familiar with the rates named in the standard as well as other commonly used measures (e.g., kilowatt hours, foot-pounds, acre-feet). Test questions on the CAHSEE for this standard focus on all three of its components: measures as rates, measures as products, and reasonableness of results.

Sample Test Question

The sample question requires the student to find an equivalent rate to 60 miles per hour. The correct answer is choice A. Students may reason that another way to express the given rate is 60 miles per 60 minutes, since 1 hour is equivalent to 60 minutes. The expression is $60 \text{ miles/hour} = 60 \text{ miles}/60 \text{ minutes}$ and that since $60 \div 60 = 1$, the rate is equivalent to 1 mile/minute. Students should also be encouraged to evaluate the answer choices in the question for their reasonableness as a restatement of this real-world rate.

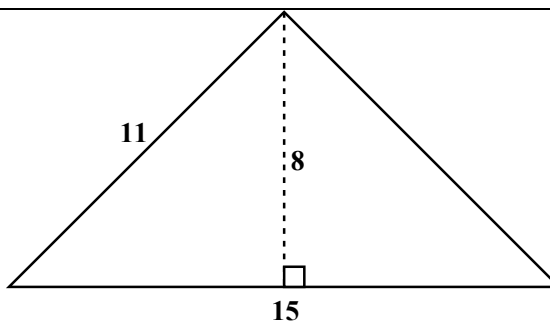
Analysis of Distractors

The distractors represent misunderstandings of the required calculations. Distractor B results from incorrectly equating 1 hour to 60 seconds. Distractor C is obtained by dividing 60 miles per hour by 10, rather than by 60. Distractor D is obtained by multiplying 60 by 6.

Strand **Measurement and Geometry (MG)**

Standard **MG2.1**
Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

Constructs **Procedural Skills, Conceptual Understanding**



What is the area of the triangle shown above?

- A 44 square units
- B 60 square units
- C 88 square units
- D 120 square units

All students should know how to compute the area and volume of basic figures and also how to apply basic formulas in many areas to solve problems. This standard requires students to find the perimeter and area of 2-dimensional figures and the surface area and volume of 3-dimensional figures. The figures tested on the CAHSEE include parallelograms, trapezoids, triangles, circles, prisms, and cylinders. Students are required to know the following formulas:

- perimeter of a polygon
- circumference of a circle
- area of a triangle
- area of a parallelogram (including rectangles)
- volume of a rectangular prism

These formulas will not be given with the test questions. The estimated value of π will be given unless π is included in the answer choices.

Sample Test Question

The sample question presents a triangle with the dimensions labeled and asks students to compute the area. The correct answer is choice B. Students should know the formula

$\frac{1}{2} \times bh$ and use it to determine that since the height of the triangle is 8 units and the base is 15 units, the area is 60 square units.

Analysis of Distractors

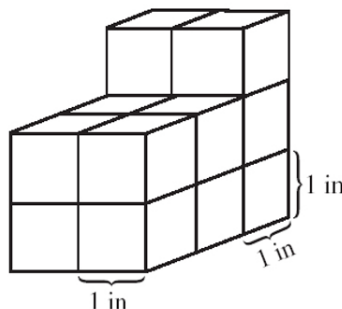
The distractors represent misapplications of the area formula. Distractor A uses 11 as the base, rather than 15. Distractor C uses 11 as the base and also fails to take $\frac{1}{2}$ of the product of the base and height. Distractor D applies the formula to the correct dimensions but contains the common error of failing to take $\frac{1}{2}$ of the product of the base and height.

Strand **Measurement and Geometry (MG)**

Standard **MG2.2**
Estimate and compute the [surface] area of more complex or irregular two-and three-dimensional figures by breaking the figures down into more basic geometric objects.

Constructs **Procedural Skills, Conceptual Understanding**

One-inch cubes are stacked as shown in the drawing below.



What is the total surface area?

- A 19 in²
- B 29 in²
- C 32 in²
- D 38 in²

This standard emphasizes the development of problem-solving skills with visual tools. CAHSEE test questions for this standard focus on both of its components. The first component requires students to identify the measurable and/or computable parts of a shape or structure. This identification task is largely dependent on the ability to visualize the familiar geometric structures that make up a more complex figure. Where estimation is required, students may be given dimensions of known shapes, scale and proportion, or grids in the visual prompt. The second component requires students to estimate and/or compute the area of the subdivided portions of a figure. Frequently the dimensions of the component parts are not given directly by labeled measurements but must be determined by such means as adding or subtracting lengths or extending lines.

Formulas will be provided where necessary, *except* for the following:

- perimeter of a polygon
- circumference of a circle
- area of a triangle
- area of a parallelogram (including rectangles)
- volume of a rectangular prism

In estimation problems, the approximate value of π that should be used to obtain the correct answer will be provided as part of the stem. Otherwise, the answer will include π (e.g., $12 + 3\pi$).

Sample Test Question

The sample question presents a drawing of an assembly of 1-inch cubes and asks students to determine the surface area. The correct answer is choice D. The foreground portion of the object contains a 2 by 2 by 2-cube grouping, and the background contains a 1 by 3-cube grouping. To find the surface area, students must visually assemble the surface of the individual cubes that make up the entire object and recognize that the object has 8 planar faces: right, rear, left, front, bottom, front top, rear face, and rear top. The surface areas of each corresponding face are 7, 6, 7, 4, 6, 4, 2, and 2 square inches, and the sum of the areas of these faces is 38 square inches.

Analysis of Distractors

The distractors represent inaccurate visualizations of the faces of the object. Distractor A includes only half of the exterior faces. Distractor B fails to include either the left or right face and either the rear face or rear top. Distractor C fails to include the bottom or rear of the figure.

Strand Measurement and Geometry (MG)

Standard MG2.3
Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.

Constructs Procedural Skills, Conceptual Understanding, Problem Solving

A cereal manufacturer needs a box that can hold twice as much cereal as the box shown below.



Which of the following changes will result in the desired box? ($V = lwh$)

- A Double the height only.
- B Double both the length and width.
- C Double both the length and height.
- D Double the length, width and height.

Scaling effects provide insights for learning advanced topics in all disciplines, and this content standard requires students to understand how changes in dimensions affect the size of objects. The standard contains two components, one addressing the effect of the scale factor on area, and the other addressing the effect of the scale factor on volume. Students should understand that when the dimensions of a two-dimensional object are multiplied by a scale factor, the resulting area is multiplied by the square of the scale factor. Students should also understand that when the dimensions of a three-dimensional object are multiplied by a scale factor, the resulting volume is multiplied by the cube of the scale factor.

Sample Test Question

The sample question presents a rectangular prism and asks students to determine the change that will double the volume of the prism. The correct answer is choice A. Students should recognize that doubling one dimension will be sufficient to double the original volume.

Analysis of Distractors

The distractors represent conceptual misunderstandings of the effect of the change of dimensions on volume. Students who choose distractors B or C may not recognize that doubling two of the dimensions will result in a new volume that is four times the original volume. Students who choose distractor D may not understand that doubling all three dimensions will result in a new volume that is eight times the original volume.

Strand	Measurement and Geometry (MG)	<p>One cubic inch is approximately equal to 16.38 cubic centimeters. Approximately how many cubic centimeters are there in 3 cubic inches?</p> <p>A 5.46</p> <p>B 13.38</p> <p>C 19.38</p> <p>D 49.14</p>
Standard	MG2.4	
Construct	Conceptual Understanding	
<p>Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or $[1 \text{ ft}^2] = [144 \text{ in}^2]$, 1 cubic inch is approximately 16.38 cubic centimeters or $[1 \text{ in}^3] = [16.38 \text{ cm}^3]$).</p>		

Unit conversions for area and volume have practical applications in students' lives. CAHSEE test questions for this standard require students to relate the changes in measurement with a change of scale to the units used and to convert between units. The emphasis is on the relationship between linear units for distance, square units for area, and cubic units for volume. Conversion formulas will be provided for linear measures except for inches to feet to yards or conversions within the metric system.

Sample Test Question

The sample question gives students an equivalence between inches and centimeters and asks for a computation based on that equivalence. The correct answer is choice D. Students should perform the computation ($16.38 \times 3 = 49.14$) or use estimation to determine the correct answer ($3 \times 18 = 48$, approximating 49.14).

Analysis of Distractors

The distractors represent incorrect choices of mathematical operations. Distractor A is obtained by dividing 16.38 by 3. Distractor B is obtained by subtracting 3 from 16.38. Distractor C is obtained by adding 3 to 16.38.

Strand

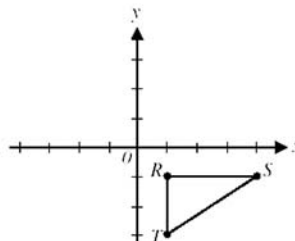
**Measurement
and Geometry
(MG)**

Standard **MG3.2**

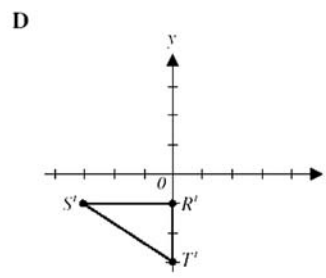
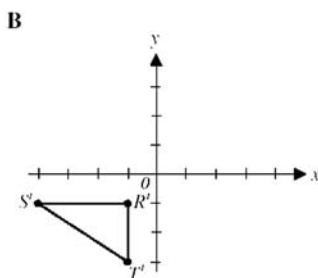
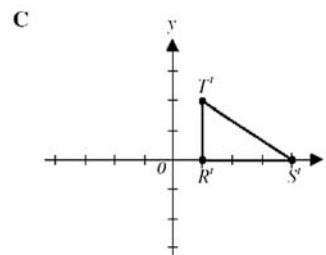
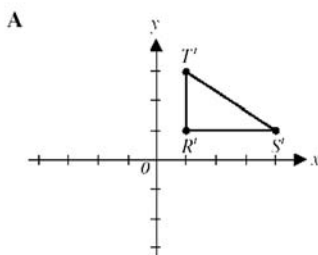
**Understand and use
coordinate graphs to
plot simple figures,
determine lengths and
areas related to them,
and determine their
image under translations
and reflections.**

Constructs

**Procedural Skills,
Conceptual
Understanding,
Problem Solving**



Which of the following triangles $R'S'T'$ is the image of triangle RST that results from reflecting triangle RST across the y -axis?



CAHSEE test questions for this standard assess students' understanding of all components of the standard: plotting with ordered pairs, determining lengths and areas from plotted figures, and finding images following transformations by translations and reflections.

Sample Test Question

The sample question requires students to choose the correct translation of an object (triangle RST) by reflecting it across the y -axis. The correct answer is choice B. Students must recognize the reflective correspondence between points R and R' , S and S' , and T and T' , and that triangle $R'S'T'$ is the reflective image, across the y -axis, of triangle RST .

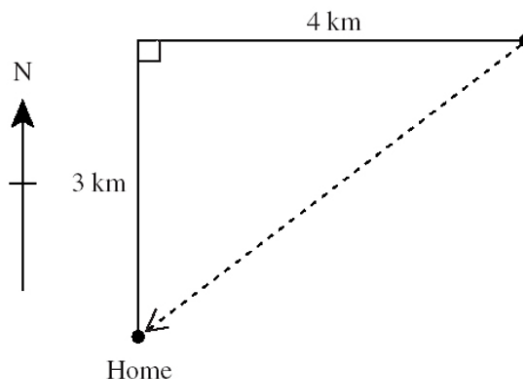
Analysis of Distractors

Distractor A is the reflection of triangle RST across the x -axis. Distractor C is the reflection of triangle RST across the x -axis and its translation by one unit down. Distractor D is the reflection of triangle RST across the y -axis and its translation by 1 unit to the right.

Strand **Measurement and Geometry (MG)**

Standard **MG3.3**
Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

Constructs **Conceptual Understanding, Problem Solving**



The club members hiked 3 kilometers north and 4 kilometers east, but then went directly home as shown by the dotted line. How far did they travel to get home?

- A 4 km
- B 5 km
- C 6 km
- D 7 km

The Pythagorean theorem is important for its problem-solving function as well as its role as a bridge between geometry and algebra. CAHSEE test questions assessing this standard address the following components of the standard: using the Pythagorean theorem to find the missing base, altitude, or hypotenuse length in a right triangle and using the Pythagorean theorem to find lengths of line segments in figures other than triangles.

Sample Test Question

The sample question presents a right triangle with base 3 km, altitude 4 km, and an unlabeled hypotenuse, in the context of a hiking trip. The correct answer is choice B. Students should understand that use of the Pythagorean theorem is appropriate to find how far the club members hiked, which is the length of the hypotenuse ($3^2 + 4^2 = c^2$). Students may also be expected to have memorized common right triangles such as this one.

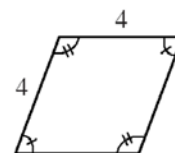
Analysis of Distractors

Distractor A is the same length as the altitude. Distractor C is an incorrect solution to the equation that applies the Pythagorean theorem. Distractor D is the sum of the base and altitude.

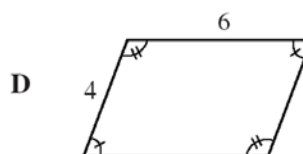
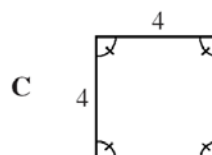
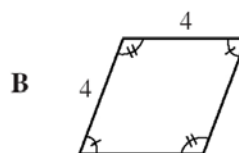
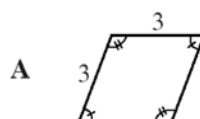
Strand **Measurement and Geometry (MG)**

Standard **MG3.4**
Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about relationships between the sides and angles of the two figures.

Construct **Conceptual Understanding**



Which figure is congruent to the figure shown above?



CAHSEE test questions for this standard require students to discriminate between figures that are or are not congruent to a given figure. Students may also be asked to identify specific reasons or other supporting information that would determine congruence between two figures. Relationships between sides and angles of figures are significant because they may indicate the presence or absence of congruence by axiomatic reasoning, such as side-angle-side congruence. Items in this standard will not involve the use of acronyms for side and angle relationships of congruent figures (e.g. SAS, SSS).

Sample Test Question

The sample question presents a figure with the dimensions of the sides given and the angles marked. Students are asked to use the characteristics of this figure to find the congruent figure among the answer choices. The correct answer is choice B. Students should understand that two polygons are congruent if all corresponding sides and angles are congruent. Reasoning from this basis, students should recognize that the figure in choice B is congruent to the given figure because of the correspondence with sides of length 4 and angles with two marks. Students should also understand that because both figures are rhombi with non-congruent adjacent angles, all four figures are congruent and the opposite angles are congruent.

Analysis of Distractors

Distractor A provides a similar, but not congruent, figure. Distractor C is a square, and distractor D is a parallelogram with non-congruent adjacent sides.

The Algebra 1 Strand

The Algebra 1 strand builds upon students' knowledge and skills developed from their experience with linear functions, tables, graphs, verbal rules, and symbolic rules. As students deepen their understanding of relations and functions, they will expand their capacity to make meaningful use of new types of functions, including polynomial, exponential, rational, and periodic functions. Important new learning includes combining functions, expressing functions in equivalent forms, and finding inverses. This experience leads to more global understanding of classes of functions as a concept and the recognition of the significant characteristics of various classes.

To demonstrate achievement in the Algebra 1 strand, students must also develop insights into mathematical abstraction and structure. Students should develop an understanding of the algebraic properties that govern the manipulation of symbols in expressions. As students become more familiar with these types of abstractions, they develop the means to solve equations and inequalities, express equivalent forms, and assert proofs.

Facility with abstraction and deeper knowledge of functions and relations give students more powerful mathematical tools to analyze and describe situations. Tools such as graphs and other visual representations of phenomena provide additional insights into problems and applications.

Standards in the Algebra 1 strand include performing operations such as opposite (additive inverse), reciprocal, and root; solving equations and inequalities with absolute values; simplifying expressions; solving multi-step problems with linear equations and inequalities; graphing linear equations and finding the x and y -intercepts; verifying points on a line given an equation; deriving linear equations; understanding and using the relationship between parallel lines and slopes; solving systems of linear equations, including meaningfully interpreting their graphical representations; performing operations and solving multi-step problems with monomials and polynomials; and solving rate, work, and percent mixture problems.

The 10 specific California Content Standards covered by the CAHSEE Algebra 1 strand are discussed in the following pages.

Strand **Algebra 1 (AI)**

Standard **AI2.0**

Students understand and use such operations as taking the opposite, finding the reciprocal, and taking a root, and raising to a fractional power. They understand and use the rules of exponents.*

Construct **Conceptual Understanding**

The perimeter, P , of a square may be found by using the formula $(\frac{1}{4})P = \sqrt{A}$, where A is the area of the square. What is the perimeter of the square with an area of 36 square inches?

- A 9 inches**
- B 12 inches**
- C 24 inches**
- D 72 inches**

Facility with inverse operations is critical for students as they solve equations and inequalities. CAHSEE test questions for this content standard focus on two of the stated components: finding the opposite (additive inverse) and finding the reciprocal. As students gain facility in algebraic reasoning, they should recognize the usefulness of finding the additive inverse and reciprocal in simplifying equations and inequalities.

Sample Test Question

The sample question gives students a formula for finding the perimeter of a square using the reciprocal $\frac{1}{4}P$. The correct answer is choice C. To solve the equation for P , students should substitute 36 for A and calculate that $P = 4\sqrt{36}$, or 24.

Analysis of Distractors

The distractors represent misapplications of the formula. Distractor A is obtained by dividing 36 by 4. Distractor B is obtained by taking the square root of 36 but multiplying by 2 instead of 4. Distractor D is obtained by multiplying 36×2 .

* The CAHSEE test blueprint does not include the crossed-out portion of this content standard.

Strand **Algebra 1 (AI)**

Standard **AI3.0**
Students solve equations and inequalities involving absolute values.

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**

If x is an integer, what is the solution to $|x - 3| < 1$?

- A** $\{-3\}$
- B** $\{-3, -2, -1, 0, 1\}$
- C** $\{3\}$
- D** $\{-1, 0, 1, 2, 3\}$

For success in algebra, students should understand the concept of absolute value and the ways its meaning is used in solving equations and inequalities. CAHSEE test questions for this standard focus on both of its components: solving equations involving absolute values and solving inequalities involving absolute values. On the exam, the tested inequalities will involve only integers and will not use nested sets of inequalities.

Sample Test Question

In the sample question, students are asked to solve an inequality involving absolute value. The correct answer is choice C. Students should recognize that this inequality, $|x - 3| < 1$, is equivalent to $-1 < x - 3 < 1$. By adding 3 to each member of the inequality, the following equivalence is obtained: $2 < x < 4$. Since x is an integer, the solution is 3 because 3 is the only integer greater than 2 and less than 4.

Analysis of Distractors

Distractor A is the negative of the correct answer and results from a misunderstanding of absolute value. Distractors B and D result from setting up the inequality improperly and then adding -3 to only two members of the inequality.

Strand **Algebra 1 (A1)**

Standard **AI4.0**

Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x - 5) + 4(x - 2) = 12$.

Constructs **Conceptual Understanding, Problem Solving**

Which of the following is equivalent to $4(x + 5) - 3(x + 2) = 14$?

- A** $4x + 20 - 3x - 6 = 14$
- B** $4x + 5 - 3x + 6 = 14$
- C** $4x + 5 - 3x + 2 = 14$
- D** $4x + 20 - 3x - 2 = 14$

Finding ways to simplify expressions before trying to solve an equation or inequality is a valuable problem-solving skill. CAHSEE test questions for this content standard involve both equations and inequalities and focus on simplification rather than solution.

Sample Test Question

In the sample question, students are asked to identify an equation that is equivalent to the given equation, thereby determining the first step in simplification. The correct answer is choice A. Students should understand that for the given equation the first step in simplifying is to expand the quantities in parentheses with each coefficient:

$$4x + 20 - 3x - 6 = 14.$$

Analysis of Distractors

The distractors present incorrect simplifications. Distractor B presents the failure to multiply 4 by 5 in the first set of parentheses and to obtain a product of +6 from multiplying -3 by 2. Distractor C presents the failure to multiply 4 by 5 in the first set of parentheses and -3 by 2 in the second set of parentheses. Distractor D presents the failure to multiply -3 by 2 in the second set of parentheses.

Strand **Algebra 1 (AI)**

 Standard **AI5.0**

Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**

 Solve for x .

$$5(2x - 3) - 6x < 9$$

- A $x < -1.5$
- B $x < 1.5$
- C $x < 3$
- D $x < 6$

Solving multi-step problems and word problems can help students develop problem-solving skills and enhance their ability to think algebraically. CAHSEE test questions for this standard focus on both of its components: finding solutions to linear equations and inequalities and providing justification for each step in the solution. Justification requires the identification of appropriate specific steps in the solution process that contribute to solving the equation or inequality.

Sample Test Question

The sample question presents an inequality and asks students to solve for the variable. The correct answer is choice D. Students should recognize that the inequality requires expanding the quantity in parentheses by multiplying it by the coefficient, combining like terms, adding 15 to both sides of the equation, and dividing both sides by 4 to obtain the value for x .

Analysis of Distractors

The distractors present mistakes in the solution process. Distractor A is obtained by subtracting 15 in the third step. Distractor B is obtained by subtracting 15 in the third step and dividing by -4 in the fourth step, or finding $+6$ as the sum of $15 + 9$. Distractor C is obtained by finding $8x$ as the sum of $2x$ and $-6x$, after failing to multiply 5 by 2 in the first set of parentheses.

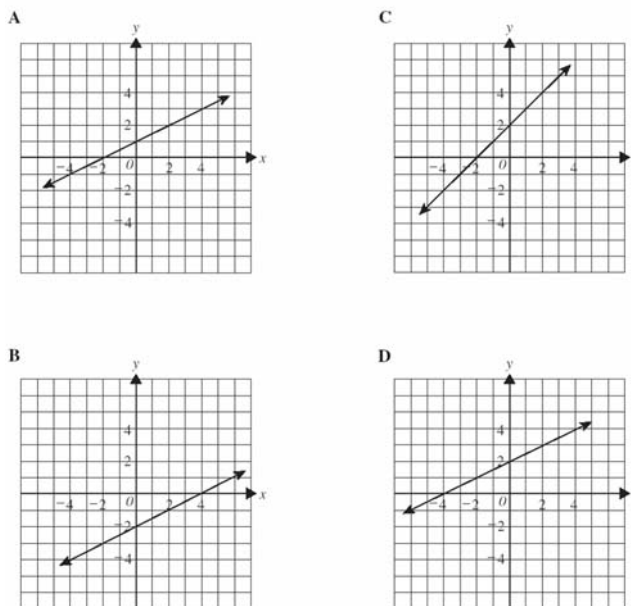
Strand Algebra 1 (AI)

Standard AI6.0

Students graph a linear equation and compute the x - and y -intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).*

Constructs Procedural Skills, Conceptual Understanding

Which of the following is the graph of $y = \frac{1}{2}x + 2$?



CAHSEE test questions for this standard focus on two components: graphing the linear equation and finding the x - and y -intercepts. Students may be asked to identify the graph that corresponds to a given equation or to identify the equation that corresponds to a given graph. The x - and y -intercepts may be identified with a single number or with ordered pairs.

Sample Test Question

The sample question presents an equation and asks students to identify the correct graph of the equation from among four choices. The correct answer is choice D. Students

should recognize that the equation, $y = \frac{1}{2}x + 2$, is in the form $y = mx + b$ and use this

information to determine the correct graph: The variable b represents the y -intercept, which in this case is 2, and the variable m represents the slope of the graph, which in this case is $\frac{1}{2}$.

Analysis of Distractors

Distractor A displays a line with a slope of $\frac{1}{2}$ and y -intercept of 1. Distractor B displays

a line with a slope of $\frac{1}{2}$ and y -intercept of -2. Distractor C displays a line with a slope of 1 and y -intercept of 2.

* The CAHSEE test blueprint does not include the crossed-out portion of this content standard because the test questions are all multiple choice.

Strand	Algebra 1 (AI)	Which of the following points lies on the line $4x + 5y = 20$? A (0, 4) B (0, 5) C (4, 5) D (5, 4)
Standard	AI7.0	
Students verify that a point lies on a line, given an equation of the line. Students are able to derive - linear equations by using the point slope formula.*		
Constructs	Procedural Skills, Conceptual Understanding, Problem Solving	

CAHSEE test questions for this standard require students to select a set of one or more points, either by ordered pairs or by graphical location, that lie on the graph of a given linear equation, or to select an equation, either by its algebraic notation or by its graph, whose graph includes one or more specified points. Components of the standard include verifying that a point lies on a given line and deriving an equation from information given about the line. To verify that points do or do not lie on a given line, students may use substitution of x or y values to find corresponding ordered pairs.

Sample Test Question

The sample question presents an equation and asks students to identify an ordered pair that would lie on the graph of that equation. The correct answer is choice A. Students should recognize that the equation $4x + 5y = 20$ represents true statements for certain real number values of x and y . The values may be tested by substituting them for x and y , respectively, in the equation. If $x = 0$ and $y = 4$, then $4(0) + 5(4) = 20$ is an equation.

Analysis of Distractors

Distractor B would mean substituting 0 for x and 5 for y , the result being that $4(0) + 5(5) = 25$ rather than 20. Distractor C means substituting 4 for x and 5 for y , the result being that $4(4) + 5(5) = 41$ rather than 20. Distractor D means substituting 5 for x and 4 for y , the result being that $4(5) + 5(4) = 40$ rather than 20.

* The CAHSEE test blueprint does not include the crossed-out portion of this content standard because the test questions are all multiple choice.

Strand **Algebra 1 (AI)**

 Standard **AI8.0**

Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. ~~Students are able to find the equation of a line perpendicular to a given line that passes through a given point.*~~

Constructs **Conceptual Understanding, Problem Solving**

What is the slope of a line parallel to the line

$$y = \frac{1}{3}x + 2?$$

 A -3

 B $-\frac{1}{3}$

 C $\frac{1}{3}$

 D 2

To demonstrate understanding of this content standard, students must know that parallel lines have equivalent slopes and different x - and y -intercepts. CAHSEE test questions for this standard may require students to find the slope of a line parallel to a given line, to identify pairs of parallel lines from their slopes, or to identify lines not parallel to a given line from a given or derived slope.

Sample Test Question

The sample question asks students to determine the slope of a line parallel to a given line. The correct answer is choice C. Students should know that parallel lines have equivalent slopes. They must also recognize that the slope of the line that is represented by the equation $y = \frac{1}{3}x + 2$ is $\frac{1}{3}$.

Analysis of Distractors

Distractor A is the negative reciprocal of the slope. Distractor B is the additive inverse of the slope. Distractor D is the y -intercept (constant).

* The CAHSEE test blueprint does not include the crossed-out portion of this content standard because the test questions are all multiple choice.

Strand **Algebra 1 (A1)**

Standard **AI9.0**

Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.

Constructs **Procedural Skills,
Conceptual
Understanding,
Problem Solving**

$$\begin{cases} 7x + 3y = -8 \\ -4x - y = 6 \end{cases}$$

What is the solution to the system of equations shown above?

- A** $(-2, -2)$
- B** $(-2, 2)$
- C** $(2, -2)$
- D** $(2, 2)$

Many real-world situations are most appropriately modeled as systems of equations, and graphs of these kinds of systems are common in newspapers and other media. CAHSEE test questions for this content standard focus on four components: solving systems of linear equations, interpreting the solutions graphically, solving a system of linear inequalities, and determining the solution sets.

Sample Test Question

The sample question presents a system of equations and asks student to find the solution. The correct answer is choice B. To solve this sample problem, students should use a method such as the following:

(multiply the second equation by 3)	$-12x - 3y = 18$
(add to the first equation)	$-5x = 10$
(divide by 5)	$x = -2$
(substitute $x = -2$ in the first equation)	$7(-2) + 3y = -8$
(expand)	$-14 + 3y = -8$
(combine)	$3y = 6$
(divide by 3)	$y = 2$

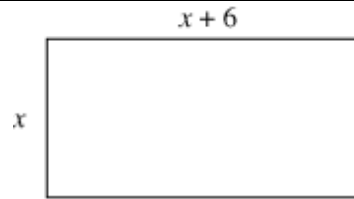
Analysis of Distractors

Distractor A results from errors in substitution and/or computation, as does distractor D. Distractor C presents a misunderstanding of the correct order of the x and y values.

Strand **Algebra 1 (AI)**

Standard **AI10.0**
Students add, subtract, multiply, and divide monomials and polynomials. Students solve multi-step problems, including word problems, by using these techniques.

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**



The length of the rectangle above is 6 units longer than the width. Which expression could be used to represent the area of the rectangle?

- A $x^2 + 6x$
- B $x^2 - 36$
- C $x^2 + 6x + 6$
- D $x^2 + 12x + 36$

This standard requires students to accurately execute arithmetic operations on monomials and polynomials and to select and use these techniques to solve problems.

Sample Test Question

The sample question presents a rectangle with length and width labeled with a monomial and a binomial, and students are asked to determine the expression that would correctly represent the area. The correct answer is choice A. Students must know to multiply the length by the width in order to find the area and then perform the computation correctly.

Analysis of Distractors

Distractors B, C, and D represent examples of incorrect multiplication of the monomial by the binomial.

Strand **Algebra 1 (AI)**

Standard **AI15.0**
Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**

Mr. Jacobs can correct 150 quizzes in 50 minutes. His student aide can correct 150 quizzes in 75 minutes. Working together, how many minutes will it take them to correct 150 quizzes?

- A 30**
- B 60**
- C 63**
- D 125**

The key assessment of student learning in any subject is whether or not students can apply their knowledge and skills to a new problem situation. CAHSEE test questions written for this standard focus on students' ability to apply their mathematical skills and knowledge to solve rate problems, work problems, and percent mixture problems.

Sample Test Question

The sample question presents a rate problem in the context of grading quizzes. The correct answer is choice A. Students should recognize that correcting 150 quizzes in 50 minutes is equivalent to the rate of 3 quizzes per minute and that correcting 150 quizzes in 75 minutes is equivalent to the rate of 2 quizzes per minute. Working together, Mr. Jacobs and his aide can correct 5 quizzes each minute:

$$150 \text{ quizzes} \div 5 \text{ quizzes/minute} = 30 \text{ minutes.}$$

Analysis of Distractors

The distractors represent misunderstandings of a rate problem. Distractor B represents the number of minutes it would take both individuals to complete the task at the rate of 2.5 quizzes/minute—the average of their rate. Distractor C is the approximate average of their time for 150 quizzes. Distractor D is the sum of the number of minutes each takes to correct 150 quizzes (50 + 75).

The Mathematical Reasoning Strand

Reasoning is an integral part of mathematics and requires several important skills, including examining patterns for regularities, making and testing conjectures about generalizations, and using formal inductive and deductive reasoning to formulate mathematical arguments. Mastery of each of these elements of reasoning requires students to work with diverse problems and activities.

Activities in this strand require language with sufficient precision, clarity, and appropriateness to support rigorous thinking. Standards in mathematical reasoning require students to analyze problems by identifying relationships, to formulate and justify conjectures, to use estimation on the basis of numerical or graphical information, to use inductive and deductive reasoning, to evaluate the reasonableness of solutions, and to generalize results and apply them to new problems.

Each question in this standard also is classified within one of the other five mathematical strands for purposes of reporting student scores.

The seven specific California Content Standards covered by the CAHSEE Mathematical Reasoning strand are discussed in the following pages.

Strand **Mathematical Reasoning (MR)**

Standard **MR1.1**
Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**

Chris drove 100 kilometers from San Francisco to Santa Cruz in 2 hours and 30 minutes. What computation will give Chris' average speed, in kilometers per hour?

- A Divide 100 by 2.5.
- B Divide 100 by 2.3.
- C Multiply 100 by 2.5.
- D Multiply 100 by 2.3.

Students must be able to analyze situations to clarify a problem and to identify those elements that will make it possible to solve the problem. CAHSEE test questions for this standard emphasize the analysis of problems rather than their solutions. The components of the standard include the following: determining relationships, discriminating between relevant and irrelevant information, identifying missing information, sequencing and prioritizing information, and observing and identifying algebraic and geometric patterns.

Sample Test Question

The sample question requires students to understand the relationship between the given distance (100 km), the given time (2 hours 30 minutes), and the rate (unknown). The correct answer is choice A. Students must recognize that rate, Chris' average speed, is the unknown variable and must know how to use the distance, rate, time equation to determine the rate. This question is also classified in the Algebra and Functions strand for purposes of reporting student scores.

Analysis of Distractors

The distractors represent errors in the application of the given information and the equation. Distractor B indicates that division is the appropriate computation; however, it provides 2.3 as an incorrect value for 2 hours and 30 minutes. Distractor C presents an incorrect operation, multiplication, although the expression for the number of hours is correct. Distractor D presents multiplication as the operation and also presents an incorrect value for 2 hours and 30 minutes.

Strand	Mathematical Reasoning (MR)	<p>If n is any odd number, which of the following is true about $n + 1$?</p> <p>A It is an odd number.</p> <p>B It is an even number.</p> <p>C It is a prime number.</p> <p>D It is the same number as $n - 1$.</p>
Standard	MR1.2	
	Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.	
Constructs	Conceptual Understanding, Problem Solving	

The challenge for many students in formulating mathematical conjectures is precision of language. CAHSEE test questions for this standard focus on both of its components: formulation of a conjecture and justification of a conjecture. Students may be asked to make conjectures based on indirect or incomplete evidence. Test questions may state a conjecture and ask students to choose among reasons that the conjecture is reasonable.

Sample Test Question

The sample question presents n as any odd number and then requires evaluation of four conjectures. The correct answer is choice B. Students should reason that if n is odd, $n + 1$ is necessarily even. Students may also attempt to find counterexamples, i.e., examples of odd n and also odd $n + 1$. This question is also classified in the Algebra and Functions strand for purposes of reporting student scores.

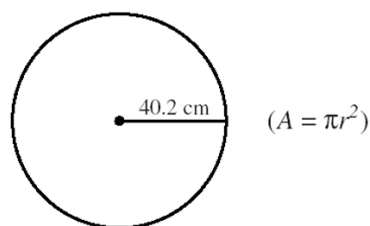
Analysis of Distractors

Students should reason that distractor A is always false because $n + 1$ must always be even. Distractor C is true only when $n = 1$, since 2 is the only even prime. Distractor D is never true, since $n - 1$ always differs from $n + 1$ by 2.

Strand **Mathematical Reasoning (MR)**

Standard **MR2.1**
Use estimation to verify the reasonableness of calculated results.

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**



Louis calculated the area of the circle above and got an answer of 50.769 cm^2 . He knew his answer was wrong because the correct answer should be about

- A $4 \times 4 \times 4 = 64$.
- B $3 \times 3 \times 40 = 360$.
- C $31 \times 4 \times 4 = 496$.
- D $3 \times 40 \times 40 = 4800$.

Students who have developed the mental habit of estimating and verifying the reasonableness of calculated results will be well prepared to respond to test questions written for standard MR2.1. This standard requires students to use estimating skills in computation and compare estimated results to calculated results in order to judge their reasonableness.

Sample Test Question

The sample test question contains the calculated result 50.769 cm^2 , and students must choose an estimation strategy that appropriately uses the dimensions of the circle and calculation for the area of a circle. The correct answer is choice D; since $A = \pi r^2$, $\pi \approx 3$, and $40.2 \approx 40$, the appropriate estimate is $3 \times 40 \times 40$. This question is also classified in the Measurement and Geometry strand for purposes of reporting student scores.

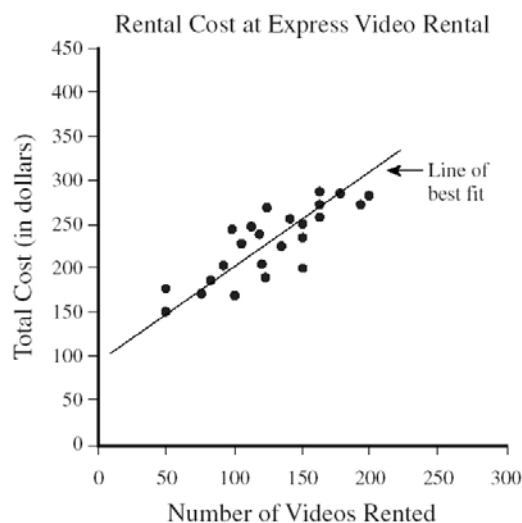
Analysis of Distractors

Distractor A represents an inappropriate selection of estimates for the dimensions of the circle. Distractor B uses the square of π , rather than the radius, as part of the estimate. Distractor C uses 4, rather than 3, as an estimate for π .

Strand **Mathematical Reasoning (MR)**

Standard **MR2.3**
Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**



Using the line of best fit shown on the scatter plot above, which of the following best approximates the rental cost per video to rent 300 videos?

- A \$3.00
- B \$2.50
- C \$2.00
- D \$1.50

Graphs provide a quick summary of data or functions but may not include the specific information required to answer a particular question. By identifying trends and patterns and using interpolation and extrapolation, students may be able to obtain a reasonable estimate of the needed information. This content standard has two components: estimating graphically and solving for unknown quantities. CAHSEE test questions for the standard may ask students to find or identify the most accurate line of best fit through a scatterplot, to extract information from a graph by interpolation or extrapolation, or to identify an equation that could be used to solve a problem shown in a graph.

Sample Test Question

The sample question includes a scatterplot that relates total cost to number of videos rented. Finding the line of best fit requires an approximation of the correspondence between total cost (y-axis) and videos rented (x-axis). The correct answer is choice D. Students should recognize that extending the line to include an x-value that corresponds to 300 videos allows the interpretation of a corresponding value of the total cost on the y-axis. From the line of best fit, renting 300 videos corresponds to approximately \$450, or \$1.50 per video. This question is also classified in the Probability and Statistics strand for purposes of reporting student scores.

Analysis of Distractors

Distractor A requires a total cost of approximately \$900 to obtain a per video cost of \$3.00, which is out of the range of the line of best fit. Distractor B requires a total cost of approximately \$750 to obtain a per video cost of \$2.50, also out of the range of the line of best fit. Similarly, distractor C requires a total cost of approximately \$600 to obtain a per video cost of \$2.00, also out of the range of the line of best fit.

Strand	Mathematical Reasoning (MR)	<p>The winning number in a contest was less than 50. It was a multiple of 3, 5, and 6. What was the number?</p> <p>A 14</p> <p>B 15</p> <p>C 30</p> <p>D It cannot be determined.</p>
Standard	MR2.4	
	Make and test conjectures by using both inductive and deductive reasoning.	
Constructs	Conceptual Understanding, Problem Solving	

Being able to identify patterns (inductive reasoning) and then testing the validity of the patterns (deductive or logical reasoning) are key skills in many fields besides mathematics. This standard requires students to use inductive and deductive reasoning to make and test conjectures. CAHSEE test questions for this standard may require reasoning from general to specific, from specific to general, as well as reasoning by use of counterexample.

Sample Test Question

The sample question requires students to use mathematical reasoning to find the winning number in a contest. The correct answer is choice C. Students may begin by finding multiples of 3, 5, and 6 that are less than 50. The multiples of 3 that are less than 50 are 3, 6, 9,...48. The multiples of 5 that are less than 50 are 5, 10, 15,...45. The multiples of 6 that are less than 50 are 6, 12, 18,...48. Thus students should recognize that the only common multiple of 3, 5, and 6 that is less than 50 is 30. This question is also classified in the Number Sense strand for purposes of reporting student scores.

Analysis of Distractors

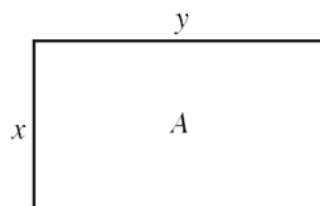
Distractor A is the sum of 3, 5, and 6, and distractor B is not a multiple of 6. Students who do not understand the concept of multiples may choose distractor D as the correct answer.

Strand **Mathematical Reasoning (MR)**

Standard **MR3.1**
Evaluate the reasonableness of the solution in the context of the original situation.

Construct **Problem Solving**

The rectangle shown below has width x , length y , and area A .



If $x = 10$ and $y > 17$, which of the following cannot be the area of the rectangle?

- A 170
- B 180
- C 190
- D 200

Checking the reasonableness of a solution is an important perspective for students, since calculators and computers are widely available for much routine computation. Content in this standard may include units, signs, or scale in relation to the magnitude described in the problem. Students are asked to determine the reasonableness of answers based on judgments about the characteristics of the quantities within mathematical problems.

Sample Test Question

The sample question presents a rectangle of dimensions 10 and more than 17 units ($y > 17$). The correct answer is choice A. Students should recognize that the area of the rectangle must be more than 170 units, because $10 \times 17 = 170$ because if the area were equal to 170, the value of y would be equal to 17, which contradicts $y > 17$. This question is also classified in the Measurement and Geometry strand for purposes of reporting student scores.

Analysis of Distractors

Distractors B, C, and D could each be the area of the rectangle, because $y = 18$, $y = 19$, and $y = 20$ do not contradict $y > 17$.

Strand	Mathematical Reasoning (MR)
Standard	MR3.3
	Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.
Construct	Problem Solving

Len runs a mile in 8 minutes. At this rate how long will it take him to run a 26-mile marathon?

Which of the following problems can be solved using the same arithmetic operations that are used to solve the problem above?

- A** Len runs 26 miles in 220 minutes. How long does it take him to run each mile?
- B** A librarian has 356 books to place on 18 shelves. Each shelf will contain the same number of books. How many books can the librarian place on each shelf?
- C** A cracker box weighs 200 grams. What is the weight of 100 boxes?
- D** Each basket of strawberries weighs 60 grams. How many baskets can be filled from 500 grams of strawberries?

One key problem-solving skill is to recognize how a new problem is like a simpler or more familiar problem. This content standard requires students to understand the process by which problems are reasoned, analyzed, and solved. CAHSEE test questions for this standard may include the selection of appropriate analogs to a given problem situation, and the relevance of the analogs as applied to the types of reasoning, patterns of operations, or logical extensions, rather than to context or other more superficial characteristics of the problem. Test questions for this standard may or may not require numerical solutions.

Sample Test Question

The sample test question presents a mathematical problem in context that requires multiplication to find a total. Students must select the most appropriate analog to finding the total time for running 26 miles at the rate of 1 mile per 8 minutes. The correct answer is choice C. Students should recognize the analog in using multiplication to determine the total weight of 100 cracker boxes, with 1 cracker box weighing 200 grams. This question is also classified in the Number Sense strand for purposes of reporting student scores.

Analysis of Distractors

The distractors present division problems. In distractor A, time, 220 minutes, is divided by distance, 26 miles. In distractor B, the number of books, 356, is divided by the number of shelves, 18. In distractor D, the total weight, 500 grams, is divided by the weight of 1 basket, 60 grams.

Section 4

Suggested Process for Using the CAHSEE to Increase Student Achievement

A Dynamic Process

Implementing standards-based instruction is a dynamic process that utilizes resources as they become available. One such resource is the released test questions from the CAHSEE. Another is the information contained in this document. The suggested process that follows will assist teachers in using the released questions and other resources to analyze their instructional program. A goal for this document is to identify where schools or districts are on the continuum of implementing standards-based instruction in mathematics and help teachers move forward in that activity. It is expected that schools will use the following material to refine and adjust these activities as appropriate to their own instructional programs.

Generating answers to the following questions will begin the process:

- What are the mathematical concepts being assessed by the CAHSEE?
- Where does this content fit in the district's standards-based instructional program?
- Are there areas being assessed that need to be added to the program?

To answer these questions, a five-step process is suggested. The following is a summary of the process with key questions for each step. A worksheet for this process is found in Appendix B.

Step One

Use the CAHSEE released questions and this Teacher Guide to identify the mathematics concepts being assessed.

- What are the mathematics standards being assessed?
- What content, knowledge, and skills of the content standards should students experience to be successful with the CAHSEE?

Step Two

See the CAHSEE in perspective with the California mathematics content standards and other statewide assessments.

- How do the standards assessed on the California Standards Tests (CST) compare with those on the CAHSEE?

- How do the tests relate to each other in terms of the standards assessed?
- How do the standards assessed by the California Standards Tests support and reinforce those standards on the CAHSEE?
- How do the constructs and foundational skills for the standards assessed on the CAHSEE inform instruction and curriculum?
- What is the mathematics for which students are being held accountable on these assessments?

Step Three

Take a closer look at instructional issues revealed through this process.

- Where are the content, knowledge, and skills of the standards to be assessed being introduced in the curriculum?
- Where are the content, knowledge, and skills of the standards to be assessed being practiced, remediated, and reinforced?
- What other content areas (science, etc.) apply, review, and practice these standards?

Step Four

Use the CAHSEE school and student data for curricular and instructional planning.

- What areas of strength are indicated by this information?
- What areas of weakness are indicated by this information?
- Are there curricular areas where individual students need further opportunities to practice, review, and/or remediate their mathematics content, knowledge, and skills?

Step Five

Use issues identified in steps one, two, three, and four to begin the process of program planning and implementation.

- How will you ensure that all students have multiple opportunities to learn, practice, remediate, and reinforce these content standards?
- How will you regularly assess students to ensure their success?

Step One

Use the CAHSEE Released Questions to Identify Mathematics Concepts Being Assessed.

Purpose

The goal of this first step is to use the released questions to develop an awareness of the mathematics concepts being assessed by the CAHSEE. Individual teachers, mathematics departments, districts, or larger groups may use this process. It is important to adjust this process, depending on the size of the group, in order to cover the full range of mathematics content of the CAHSE test questions. A minimum of 20 test questions must be reviewed in order to gain maximum benefits.

Materials

- Local curriculum
- Released questions from the CAHSEE
- CAHSEE blueprint (Appendix A)
- CAHSEE Teacher Guide

Activity

This activity is for a group of at least 20 participants. The process will work, however, with an individual teacher or group of teachers. A discussion of the questions by a group of several math teachers will be richer than one with only a few teachers, but the efforts of the small group will also lead them to the goal.

A. Divide into groups of four to five people.

- 1) Have each group work through five test questions from the released CAHSEE questions, using a collaborative process.
- 2) Based on the released items just reviewed, identify the mathematics content being assessed by each question.
- 3) Identify the content, knowledge, and skills should students experience to be successful with the CAHSEE.

B. Share the discussion from each small group with the total group. The broad spectrum of mathematics content covered by these items should be identified.

Step Two

See the CAHSEE in Perspective with the California Mathematics Content Standards and Other Statewide Assessments.

Purpose

To gain a deeper understanding of how students are being held accountable for the California Mathematics Content Standards through the CAHSEE and other statewide assessments.

Materials

- Worksheet (in Appendix B)
- CAHSEE Teacher Guide
- California Mathematics Content Standards
- Available information about other statewide assessments (CAT/6, GSE, MDTP, and ELM)

Activity

- A. Divide into groups of four to five people. Discuss various assessments—tools, type, source, purpose, grades, and when tested—with the total group: CAHSEE, the CAT/6, California Standards Tests (CST), Golden State Examinations (GSE), Mathematics Diagnostic Testing Program (MDTP), and the Entry Level Mathematics test (ELM). Discuss the following questions:
 - How do the standards assessed on the CST compare with those on the CAHSEE?
 - How do the tests relate to each other in terms of the standards assessed?
 - How do the standards assessed by the CST support and reinforce those standards on the CAHSEE?
 - What is the mathematics content for which students are being held accountable on these assessments?
- B. Examine the sample questions and discussions of the content standards contained in this Teacher Guide.
 - What are the components and foundational skills in the identified standards?
 - Which component of the standards do the sample questions address?
- C. In small groups, use the test questions reviewed in step one and in the Teacher Guide to discuss the following questions:
 - Which standards are assessed by each of the questions reviewed? List each standard on the worksheet found in Appendix B.
 - What component(s) and foundational skills for the identified standards were addressed in each item?
 - What additional components might be assessed on the CAHSEE for this standard?

D. Continue in small groups to:

- Write questions for each component of at least two standards assessed by the CAHSEE. (Teachers are encouraged to continue this process with other standards as they work with the California Mathematics Content Standards throughout the year.)
- Fill in the first two rows of the worksheet in Appendix B as more standards and items are reviewed.
- Determine the multiple-choice options. (It is not necessary to have all options for multiple-choice items in the beginning of this process. When students are given the questions, the possible errors may be generated). These multiple-choice questions can be used for review of the content with students.

E. Finish the small group activity with a discussion of additional questions:

- How do the components and foundational skills for these standards inform instruction and curriculum?
- How are the components and foundational skills supported and reinforced by the California Standards Tests?

F. Share small group insights with the total group.

NOTE: Many teachers find the level of the standards being assessed in CAHSEE lower than the high school level. However, these standards are foundational skills for all higher-level mathematics courses. This fact indicates that the curriculum at all grade levels needs to be adjusted to help students attain more comprehensive mathematics at all grade levels. Other California standards at the high school level are being assessed by the California Standards Tests and the Golden State Exams.

Step Three

Take a Closer Look at Instructional Issues Revealed Through this Process.

Purpose

To identify what is currently being taught in comparison to what students should be taught for achievement on the CAHSEE and the California Mathematics Content Standards.

Materials

- Released CAHSEE test questions
- CAHSEE blueprint (Appendix A)
- Local curriculum
- Worksheet (in Appendix B)
- California Mathematics Content Standards and/or California Mathematics Framework

Activity

Use the California Mathematics Content Standards or the California Mathematics Framework for this step of the process.

Based on the problems reviewed, the content discussed, and the state content standards identified, consider the standards for which all students are being held accountable and discuss related curricular issues. Use the worksheet in Appendix B to document this discussion. The following questions may be helpful:

- At what grade level are the content, knowledge, and skills being introduced in the curriculum?
- At what grade level are the content, knowledge, and skills being practiced, remediated, and/or reinforced?
- What are other content areas (science, etc.) that use applications where these mathematics standards are being reviewed and practiced?

Teachers should recognize that reviewing, reinforcing, and/or remediating are ongoing activities.

Step Four

Use the CAHSEE school and student data for curricular and instructional planning.

Purpose

To identify areas of strength and weakness for school and individual students related to curriculum and instruction.

Materials

- School Report from the most recent CAHSEE administration
- Student and Parent Report from the most recent CAHSEE administration
- Data on the released test questions obtained from the CAHSEE website

Activity

- A. Review the School Report. It shows:
 - The mean scale score (between 250 and 450)
 - The number and percent of students who passed (at or above scale score of 350)
 - The average percent correct for all students
 - The average percent correct for all students by mathematical strand
- B. Review the Student Report. It shows:
 - The student's scale score (between 250 and 450)
 - Pass or no pass
 - Number of questions, number correct, and percent correct for each strand
- C. Review the data (p-values) on the released test questions and the corresponding data for the district and school.
- D. While analyzing the data, ask the following questions to give curricular and instructional guidance, using the worksheet in Appendix B to document the discussion:
 - What areas of strength are indicated by this information?
 - What areas of weakness are indicated by this information?
 - What are areas of the curriculum where individual students need further opportunities to review, reinforce, and/or remediate their mathematics knowledge and skills?

Step Five

Use issues identified in steps one, two, three, and four to begin the process of program planning and implementation.

Purpose

To apply the information gathered through this process to develop specific strategies for increasing student achievement with the CAHSEE and the California Mathematics Content Standards.

Materials

- Local curriculum
- CAHSEE released test questions
- CAHSEE blueprint (Appendix A)
- CAHSEE Teacher Guide
- Completed worksheet (in Appendix B)
- Planning tool (in Appendix B)

Activity

Using the completed worksheet from Appendix B, begin planning how the district, school, or teacher will ensure that the necessary mathematics content is accessible to all students. Consider the following:

- How will you ensure that all students have multiple opportunities to learn, practice, remediate, and reinforce the content standards?
- Use the planning tool in Appendix B to help develop your district, school, and/or classroom plan. The first row has been filled in as a model.

Appendix A

CAHSEE Mathematics Blueprint*

California Content Standard	Number of Items
Grade 6 — Statistics, Data Analysis, and Probability	6 Items Total
1.0 Students compute and analyze statistical measurements for data sets:	
1.1 Compute the range, mean, median, and mode of data sets.	1
1.2 Understand how additional data added to data sets may affect these computations of measures of central tendency.	0
1.3 Understand how the inclusion or exclusion of outliers affect measures of central tendency.	0
1.4 Know why a specific measure of central tendency (mean, median, mode) provides the most useful information in a given context.	0
2.0 Students use data samples of a population and describe the characteristics and limitations of the samples:	
2.1 Compare different samples of a population with the data from the entire population and identify a situation in which it makes sense to use a sample.	0
2.2 Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.	0
2.3 Analyze data displays and explain why the way in which the question was asked might have influenced the results obtained and why the way in which the results were displayed might have influenced the conclusions reached.	0
2.4 Identify data that represent sampling errors and explain why the sample (and the display) might be biased.	0
2.5 Identify claims based on statistical data and, in simple cases, evaluate the validity of the claims.	1
3.0 Students determine theoretical and experimental probabilities and use these to make predictions about events:	
3.1 Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome.	1
3.2 Use data to estimate the probability of future events (e.g., batting averages or number of accidents per mile driven).	0
3.3 Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if P is the probability of an event, 1-P is the probability of an event not occurring.	2
3.4 Understand that the probability of either of two disjoint events occurring is the sum of the two individual probabilities and that the probability of one event following another, in independent trials, is the product of the two probabilities.	0
3.5 Understand the difference between independent and dependent events.	1
Grade 7 — Number Sense	14 Items Total
1.0 Students know the properties of, and compute with, rational numbers expressed in a variety of forms:	
1.1 Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation.	1
1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.	3
1.3 Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.	2
1.4 Differentiate between rational and irrational numbers.	0
1.5 Know that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions.	0
1.6 Calculate the percentage of increases and decreases of a quantity.	1
1.7 Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.	2

* Approved by the State Board of Education on December 7, 2000

CAHSEE Mathematics Blueprint*

California Content Standard	Number of Items
Grade 7 — Number Sense, continued	
2.0 Students use exponents, powers, and roots and use exponents in working with fractions:	
2.1 Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.	1
2.2 Add and subtract fractions by using factoring to find common denominators.	1
2.3 Multiply, divide, and simplify rational numbers by using exponent rules.	1
2.4 Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.	1
2.5 Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.	1
Grade 7 — Algebra and Functions	17 Items Total
1.0 Students express quantitative relationships by using algebraic terminology, expressions, equations, inequalities, and graphs:	
1.1 Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).	2
1.1 Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$.	1
1.3 Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.	0
1.4 Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.	0
1.5 Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.	3
2.0 Students interpret and evaluate expressions involving integer powers and simple roots:	
2.1 Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.	1
2.2 Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.	1
3.0 Students graph and interpret linear and some nonlinear functions:	
3.1 Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.	1
3.2 Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and an equilateral triangle base of varying lengths).	0
3.3 Graph linear functions, noting that the vertical change (change in y-value) per unit of horizontal change (change in x-value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.	2
3.4 Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of a line equals the quantities.	1
4.0 Students solve simple linear equations and inequalities over the rational numbers:	
4.1 Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.	3
4.2 Solve multi-step problems involving rate, average speed, distance, and time or a direct variation.	2

* Approved by the State Board of Education on December 7, 2000

CAHSEE Mathematics Blueprint*

California Content Standard	Number of Items
Grade 7 — Measurement and Geometry	17 Items Total
1.0 Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems:	
1.1 Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).	2
1.2 Construct and read drawings and models made to scale.	1
1.3 Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.	2
2.0 Students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area, and volume are affected by changes of scale:	
2.1 Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelogram, trapezoids, squares, triangles, circles, prisms and cylinders.	3
2.2 Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.	2
2.3 Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and volume is multiplied by the cube of the scale factor.	1
2.4 Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or $[1 \text{ ft}^2] = [144 \text{ in}^2]$, 1 cubic inch is approximately 16.38 cubic centimeters or $[1 \text{ in}^3] = [16.38 \text{ cm}^3]$.)	1
3.0 Students know the Pythagorean theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures:	
3.1 Identify and construct basic elements of geometric figures (e.g., altitudes, midpoints, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.	0
3.2 Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.	2
3.3 Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.	2
3.4 Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.	1
3.5 Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.	0
3.6 Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).	0
Grade 7 — Statistics, Data Analysis, and Probability	6 Items Total
1.0 Students collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set by hand and through the use of an electronic spreadsheet software program:	
1.2 Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.	2
1.2 Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).	2
1.3 Understand the meaning of, and be able to compute the minimum, the lower quartile, the median, the upper quartile, and the maximum of a data set.	2

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CAHSEE Mathematics Blueprint*

California Content Standard	Number of Items
Grade 7 — Mathematical Reasoning	8 Items Total Plus Integrated into Other Strands
1.0 Students make decisions about how to approach problems:	
1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.	2
1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.	1
1.3 Determine when and how to break a problem into simpler parts.	0
2.0 Students use strategies, skills, and concepts in finding solutions:	
2.1 Use estimation to verify the reasonableness of calculated results.	1
2.2 Apply strategies and results from simpler problems to more complex problems.	0
2.3 Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.	1
2.4 Make and test conjectures by using both inductive and deductive reasoning.	1
2.5 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.	0
2.6 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.	0
2.7 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.	0
2.8 Make precise calculations and check the validity of the results from the context of the problem.	0
3.0 Students determine a solution is complete and move beyond a particular problem by generalizing to other situations:	
3.1 Evaluate the reasonableness of the solution in the context of the original situation.	1
3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.	0
3.3 Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.	1
Algebra I	12 Items Total
1.0 Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable:	
1.1 Students use properties of numbers to demonstrate whether assertions are true or false.	0
2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, and taking a root, and raising to a fractional power. They understand and use the rules of exponents.	1
3.0 Students solve equations and inequalities involving absolute values.	1
4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x - 5) + 4(x - 2) = 12$.	2
5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.	1

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CAHSEE Mathematics Blueprint*

California Content Standard	Number of Items
Algebra I, continued	
6.0 Students graph a linear equation and compute the x- and y-intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).	2 (1 graphing item; 1 computing item)
7.0 Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations. by using the point-slope formula.	1
8.0. Students understand the concepts of parallel lines and perpendicular lines and how their slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.	1
9.0 Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.	1
10.0 Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.	1
11.0 Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.	0
12.0 Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.	0
13.0 Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.	0
14.0 Students solve a quadratic equation by factoring or completing the square.	0
15.0 Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.	1
16.0 Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.	0
17.0 Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.	0
18.0 Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.	0
19.0 Students know the quadratic formula and are familiar with its proof by completing the square.	0
20.0 Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.	0
21.0 Students graph quadratic functions and know that their roots are the x-intercepts.	0
22.0 Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.	0
23.0 Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.	0
24.0 Students use and know simple aspects of a logical argument:	
24.1 Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.	0
24.2 Students identify the hypothesis and conclusion in logical deduction.	0
24.3 Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion.	0
25.0 Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements:	
25.1 Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.	0
25.2 Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step.	0
25.3 Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.	0
TOTAL	80

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Appendix B

California High School Exit Examination (CAHSEE) Worksheet

	Number Sense	Algebra and Functions, Algebra 1	Measurement and Geometry	Statistics, Data Analysis, and Probability	Mathematical Reasoning
Step Two Released Questions: Which standards are covered?					
Other standards to consider: Which additional standards need to be covered?					
Step Three What content, knowledge, and skills are students currently experiencing? What grade level are the standards?					

California High School Exit Examination (CAHSEE) Worksheet

	Number Sense	Algebra and Functions, Algebra 1	Measurement and Geometry	Statistics, Data Analysis, and Probability	Mathematical Reasoning
What content, knowledge, and skills should students experience to be successful with the CAHSEE?					
Step Four What areas of strength and/or weakness are indicated by the data?					
What areas need more focus for student success?					

California High School Exit Examination (CAHSEE) Planning Tool

What?	Who? (district/school/teacher)	When?	How? (resources)
CAHSEE Teacher Guide Section 4 (EXAMPLE)	Classroom teachers	Use throughout the school year to prepare for teaching in a standards-based classroom that moves toward success on the CAHSEE.	Use a professional development tool. Determine the components of the standards and teach students the mathematics concepts in a way that will help them be successful in demonstrating proficiency on a standards-based assessment.